Differential Equations

Question1

The population p of the city at time t is given by $\frac{dp}{dt} = \frac{p}{2} - 100$.If initial population is 100 then p = MHT CET 2025 (5 May Shift 2)

Options:

A. $200 + 100e^{\frac{t}{2}}$

B. $200 - 100e^{\frac{t}{2}}$

C. $300 - 100e^{\frac{t}{2}}$

D. $300 + 100e^{\frac{t}{2}}$

Answer: B

Solution:

Solve $\frac{dp}{dt} = \frac{p}{2} - 100$.

This linear ODE has equilibrium $p = 200$. Write

$$\frac{dp}{dt} - \frac{1}{2}p = -100.$$

General solution:

$$p(t) = 200 + Ce^{t/2}.$$

Use $p(0) = 100$:

$$100 = 200 + C \Rightarrow C = -100.$$

$$\boxed{p(t) = 200 - 100\,e^{t/2}}$$

Question2

The solution of the equation $\frac{\mathrm{dy}}{\mathrm{d}x} = \frac{1}{x+\mathrm{y}+1}$ is MHT CET 2025 (5 May Shift 2)

Options:

A. $x = \log(x + \mathrm{y} + 2) + \mathrm{c}$, where c is the constant of integration

B. $x = \log(x + \mathrm{y} - 2) + \mathrm{c}$, where c is the constant of integration

C. $\mathrm{y} = \log(x + \mathrm{y} + 2) + \mathrm{c}$, where c is the constant of integration

D. $\mathrm{y} = \log(x + \mathrm{y} - 2) + \mathrm{c}$, where c is the constant of integration

Answer: C

Solution:

Solve

$$\frac{dy}{dx} = \frac{1}{x+y+1}.$$

Notice

$$\frac{d}{dx}\ln(x+y+2) = \frac{1+y'}{x+y+2}.$$

Set $y' = \dfrac{1+y'}{x+y+2} \Rightarrow y'(x+y+1) = 1 \Rightarrow y' = \dfrac{1}{x+y+1}$,
which matches the DE. Therefore integrate both sides:

$$\int y'\,dx = \int \frac{d}{dx}\ln(x+y+2)\,dx$$

$$\Rightarrow \quad y = \ln(x+y+2) + C.$$

So the solution is

$$\boxed{y = \log(x+y+2) + C} \qquad \text{(option C)}.$$

Question3

The solution of $\log\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) = 2\mathrm{x} - 5\mathrm{y}, \mathrm{y}(0) = 0$ **is MHT CET 2025 (5 May Shift 2)**

Options:

A. $2e^{2x} + 5e^{5y} = 6$

B. $5e^{2x} - 2e^{5y} = 3$

C. $2e^{2x} - 5e^{5y} = 6$

D. $5e^{2x} + 2e^{5y} = 3$

Answer: B

Solution:

Given $\ln\left(\frac{dy}{dx}\right) = 2x - 5y$.

$$\frac{dy}{dx} = e^{2x-5y} \quad \Rightarrow \quad e^{5y}\,dy = e^{2x}\,dx$$

Integrate:

$$\int e^{5y}\,dy = \int e^{2x}\,dx \Rightarrow \frac{1}{5}e^{5y} = \frac{1}{2}e^{2x} + C$$

Rearrange:

$$5e^{2x} - 2e^{5y} = C'.$$

Use $y(0) = 0$:

$$5e^{0} - 2e^{0} = C' \Rightarrow C' = 3.$$

$$\boxed{5e^{2x} - 2e^{5y} = 3}$$

Question4

The integrating factor of the differential equation $x\frac{\mathrm{dy}}{\mathrm{d}x} + \mathrm{y}\log x = xe^{x} \cdot x^{\frac{-1}{2}}\log x (x > 0)$ is MHT CET 2025 (5 May Shift 2)

Options:

A. $(\log x)^{x}$

B. $x^{\log x}$

C. $(\sqrt{x})^{\log x}$

D. $\mathrm{e}^{\sqrt{x}\log x}$

Answer: C

Solution:

Put the DE in linear form:

$$x\frac{dy}{dx} + y\log x = xe^{x} \cdot x^{-1/2}\log x \quad \Rightarrow \quad \frac{dy}{dx} + \frac{\log x}{x}y = e^{x}x^{-1/2}\log x.$$

Thus $P(x) = \frac{\log x}{x}$.

Integrating factor:

$$\mathrm{IF} = \exp\left(\int P(x)\,dx\right) = \exp\left(\int \frac{\log x}{x}\,dx\right) = \exp\left(\frac{(\log x)^2}{2}\right).$$

Since $\exp\left(\frac{(\log x)^2}{2}\right) = (\sqrt{x})^{\log x}$, the integrating factor is

$$\boxed{(\sqrt{x})^{\log x}} \quad \text{(option C)}.$$

Question5

The equation of a curve whose normal at any point has a slope which is the same as the ordinate and which passes through $(1, -1)$ is $2x = \mathrm{k}\left(3 - \mathrm{y}^2\right)$. Then k is MHT CET 2025 (27 Apr Shift 2)

Options:

A. 1

B. 2

C. 3

D. 4

Answer: A

Solution:

Let the curve be $y(x)$.
"Normal slope equals the ordinate" ⇒ slope of normal $= y$.
But normal slope $= -\frac{1}{y'}$. Hence

$$-\frac{1}{\frac{dy}{dx}} = y \quad \Rightarrow \quad \frac{dy}{dx} = -\frac{1}{y}.$$

Separate and integrate:

$$y\,dy = -dx \ \Rightarrow \ \frac{y^2}{2} = -x + C \ \Rightarrow \ 2x = 2C - y^2.$$

Given the curve can be written $2x = k(3 - y^2)$. Matching coefficients gives $-1 = -k \Rightarrow k = 1$.
(Then $2C = 3$ and the point $(1, -1)$ satisfies it.)

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Question6

The slope of the tangent at (x, y) to the curve passing through $(2, 1)$ is $\frac{x^2+y^2}{2xy}$, then the equation of the curve is MHT CET 2025 (27 Apr Shift 2)

Options:

A. $x(x^2 + y^2) = 10$

B.

$x(x^2 - y^2) = 6$

C.

$2(x^2 - y^2) = 6y$

D.

$2(x^2 - y^2) = 3x$

Answer: D

Solution:

$$\frac{dy}{dx} = \frac{x^2 + y^2}{2xy}$$

is homogeneous. Put $y = vx \Rightarrow y' = v + xv'$.

$$v + xv' = \frac{1 + v^2}{2v} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{1 - v^2}{2v}.$$

Separate and integrate:

$$\frac{2v}{1 - v^2}dv = \frac{dx}{x} \ \Rightarrow \ -\ln(1 - v^2) = \ln x + C \ \Rightarrow \ 1 - v^2 = \frac{A}{x}.$$

Since $v = \frac{y}{x}$,

$$\left(\frac{y}{x}\right)^2 = 1 - \frac{A}{x} \quad \Rightarrow \quad y^2 = x^2 - Ax.$$

Use $(2, 1)$: $1 = 4 - 2A \Rightarrow A = \frac{3}{2}$.

Thus

$$2(x^2 - y^2) = 3x,$$

which matches **option D.**

Question7

The order and degree of the differential equation $3 - \left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}} = \left(\frac{dy}{dx}\right)^5$ are respectively MHT CET 2025 (27 Apr Shift 2)

Options:

A. 3, 7

B. 3, 8

C. 4, 7

D. 10, 7

Answer: A

Solution:

Step 1: Simplify the differential equation

The given differential equation is:

$$3-\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}=\left(\frac{dy}{dx}\right)^5$$

To find the degree, the equation must be free from fractional powers. We can eliminate the fractional power by raising both sides to the power of 3.

$$\left[3-\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}\right]^3=\left[\left(\frac{dy}{dx}\right)^5\right]^3$$

This simplifies to:

$$\left[3-\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}\right]^3=\left(\frac{dy}{dx}\right)^{15}$$

Expanding the left side of the equation using the binomial theorem, we get:

$$3^3-3\cdot 3^2\cdot\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}+3\cdot 3\cdot\left[\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}\right]^2-\left[\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}\right]^3=\left(\frac{dy}{dx}\right)^{15}$$

$$27-27\left(\frac{d^3y}{dx^3}\right)^{\frac{7}{3}}+9\left(\frac{d^3y}{dx^3}\right)^{\frac{14}{3}}-\left(\frac{d^3y}{dx^3}\right)^{7}=\left(\frac{dy}{dx}\right)^{15}$$

The highest order derivative is $\frac{d^3y}{dx^3}$ and its highest power is 7.

Step 2: Determine the order and degree

The order of a differential equation is the order of the highest derivative present in the equation. The highest derivative in the given equation is $\frac{d^3y}{dx^3}$, which is a third-order derivative. Therefore, the order is 3.

The degree of a differential equation is the power of the highest order derivative after the equation has been made free from radicals and fractional powers. In the simplified equation, the highest power of the highest order derivative, $\frac{d^3y}{dx^3}$, is 7. Therefore, the degree is 7.

Answer:

The order and degree of the differential equation are **3 and 7**, respectively.

Question8

The equation of the curve passing through the point $(0, 2)$ given that the sum of the ordinate and abscissa of any point exceeds the slope of the tangent to the curve at that point by 5 is MHT CET 2025 (26 Apr Shift 2)

Options:

A. y $= x - 4 - 2e^x$

B. $y = 4 - x - 2e^x$

C. y $= 4 + x - 2e^x$

D. $y = 4 - x + 2e^x$

Answer: B

Solution:

Given: "sum of abscissa and ordinate exceeds the slope by 5."
So at any (x, y):

$$x + y = (\text{slope}) + 5 \Rightarrow \frac{dy}{dx} = x + y - 5.$$

Solve the linear DE $y' - y = x - 5$:

$$\text{IF} = e^{-x}, \quad (e^{-x}y)' = e^{-x}(x - 5).$$

$$e^{-x}y = \int (x - 5)e^{-x}dx = (-(x + 1) + 5)e^{-x} + C = (-x + 4)e^{-x} + C.$$

$$y = -x + 4 + Ce^{x}.$$

Use $y(0) = 2 \Rightarrow 2 = 4 + C \Rightarrow C = -2.$

$$\boxed{y = 4 - x - 2e^{x}}$$

Question9

The solution of the differential equation. $(1 + x)\frac{\mathrm{dy}}{\mathrm{d}x} - x\mathrm{y} = 1 - x$ **is MHT CET 2025 (26 Apr Shift 2)**

Options:

A. $\mathrm{y}(1 + x) = x + \mathrm{ce}^{x}$, where c is the constant of integration

B. $\mathrm{y}(1 + x) = \mathrm{ce}^{x}$, where c is the constant of integration

C. $\mathrm{y}(1 - x) = x - \mathrm{ce}^{x}$, where c is the constant of integration

D. $\mathrm{y}(1 + x) = x\ \mathrm{ce}^{-x}$, where c is the constant of integration

Answer: A

Solution:

$$(1+x)\frac{dy}{dx} - xy = 1 - x \quad \Rightarrow \quad y' - \frac{x}{1+x}y = \frac{1-x}{1+x}$$

Linear DE with $P(x) = -\frac{x}{1+x}$.

Integrating factor:

$$\mu = \exp\int P\,dx = \exp\left(-\int \frac{x}{1+x}dx\right) = \exp(-x + \ln(1+x)) = (1+x)e^{-x}.$$

Then

$$\frac{d}{dx}(\mu y) = \mu \cdot \frac{1-x}{1+x} = e^{-x}(1-x).$$

Integrate:

$$(1+x)e^{-x}y = \int e^{-x}(1-x)\,dx = xe^{-x} + C.$$

Multiply by e^x:

$$\boxed{y(1+x) = x + Ce^x}$$

Question10

The differential equation representing the family of parabolas having vertex at the origin and axis along the positive Y-axis is MHT CET 2025 (26 Apr Shift 2)

Options:

A. $x\frac{\mathrm{dy}}{\mathrm{d}x} - 2\mathrm{y} = 0$

B. $\frac{\mathrm{dy}}{\mathrm{d}x} + x\mathrm{y} = 0$

C. $x\frac{\mathrm{dy}}{\mathrm{d}x} + \mathrm{y} = 0$

D. $x^2\frac{\mathrm{dy}}{\mathrm{d}x} + \mathrm{y} = 0$

Answer: A

Solution:

For parabolas with vertex at the origin and axis along the positive y-axis, the family is

$$y = ax^2 \quad (a > 0).$$

Differentiate:

$$\frac{dy}{dx} = 2ax.$$

Eliminate the parameter a using $a = \frac{y}{x^2}$:

$$\frac{dy}{dx} = 2\frac{y}{x} \implies x\frac{dy}{dx} - 2y = 0.$$

So the differential equation is

$$\boxed{x\frac{dy}{dx} - 2y = 0}.$$

Question11

The population of towns A and B increase at the rate proportional to their population present at that time. At the end of the year 1984, the population of both the towns was 20,000 . At the end of the year 1989, the population of town A was 25,000 and that of town B was 28,000. The difference of populations of towns A and B at the end of 1994 was MHT CET 2025 (26 Apr Shift 2)

Options:

A. 5950

B. 8000

C. 7950

D. 6950

Answer: C

Solution:

Since growth is proportional to population, each town grows exponentially.

At end of 1984: $P_A = P_B = 20000$.

From 1984 → 1989 (5 years):

- $P_A(1989) = 20000 \cdot e^{5k_A} = 25000 \Rightarrow e^{5k_A} = 1.25$
- $P_B(1989) = 20000 \cdot e^{5k_B} = 28000 \Rightarrow e^{5k_B} = 1.40$

At end of 1994 (10 years after 1984):

$$P_A(1994) = 20000 \cdot (e^{5k_A})^2 = 20000 \cdot (1.25)^2 = 31250$$

$$P_B(1994) = 20000 \cdot (e^{5k_B})^2 = 20000 \cdot (1.40)^2 = 39200$$

Difference $= 39200 - 31250 = \boxed{7950}$.

Question12

The general solution of the differential equation $\frac{dy}{dx} = \cot x \cdot \cot y$ is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $\cos x = c \operatorname{cosec} y$, where c is the constant of integration.

B. $\sin x = c \sec y$, where c is the constant of integration.

C. $\sin x = x \cos y$, where c is the constant of integration.

D. $\cos x = c \sin y$, where c is the constant of integration.

Answer: B

Solution:

No Solution

$$\frac{dy}{dx} = \cot x \cdot \cot y = \frac{\cos x}{\sin x} \cdot \frac{\cos y}{\sin y}$$

Separate:

$$\frac{\sin y}{\cos y}\,dy = \frac{\cos x}{\sin x}\,dx \Rightarrow \int \tan y\,dy = \int \cot x\,dx$$

$$-\ln|\cos y| = \ln|\sin x| + C \Rightarrow \ln(\sec y) = \ln(C\sin x)$$

$$\boxed{\sin x = c\sec y}$$

Question13

The money invested in a company is compounded continuously. If ₹400 invested today becomes ₹800 in 6 years, then at the end of 30 years, it will become (in ₹) MHT CET 2025 (26 Apr Shift 1)

Options:

A. 18101.76

B. 12800

C. 9050.88

D. 12804

Answer: B

Solution:

₹12,800

With continuous compounding $A = Pe^{rt}$.

Given $800 = 400e^{6r} \Rightarrow e^{6r} = 2 \Rightarrow r = \frac{\ln 2}{6}$.

After 30 years:

$$A = 400e^{30r} = 400e^{30\cdot\frac{\ln 2}{6}} = 400e^{5\ln 2} = 400\cdot 2^5 = 400\cdot 32 = 12800.$$

Question14

The equation of a curve passing through $(1,0)$ and having slope of tangent at any point (x,y) of the curve as $\frac{\mathrm{y}-1}{x^2+x}$ is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $2(\mathrm{y}-1) + x(x+1) = 0$

B. $2x-(y-1)(x+1)=0$

C. $2x+(x+1)(y-1)=0$

D. $2x(y-1)+(x+1)=0$

Answer: C

Solution:

Answer: $2x+(x+1)(y-1)=0$

Solve the DE:

$$\frac{dy}{dx}=\frac{y-1}{x^2+x}=\frac{y-1}{x(x+1)}.$$

Separate and integrate:

$$\int\frac{dy}{y-1}=\int\frac{dx}{x(x+1)}=\int\left(\frac{1}{x}-\frac{1}{x+1}\right)dx=\ln|x|-\ln|x+1|+C.$$

So

$$\ln|y-1|=\ln\left|\frac{x}{x+1}\right|+C\ \Rightarrow\ y-1=C\frac{x}{x+1}.$$

Use $(1,0)$:

$$-1=C\cdot\frac{1}{2}\ \Rightarrow\ C=-2.$$

Thus

$$y-1=-\frac{2x}{x+1}\ \Rightarrow\ 2x+(x+1)(y-1)=0.$$

Question15

The differential equation which represents the family of curves $y=C_1e^{c_2x}$, where C_1, C_2 are arbitrary constants is MHT CET 2025 (26 Apr Shift 1)

Options:

A. $y''=y'y$

B. $yy''=y'$

C. $yy''=(y')^2$

D. $y'=y^2$

Answer: C

Solution:

Answer: $yy'' = (y')^2$ (Option C)

Let $y = C_1 e^{C_2 x}$.

$$y' = C_1 C_2 e^{C_2 x} = C_2 y, \qquad y'' = C_2 y' = C_2^2 y.$$

Eliminate C_2:

$$(y')^2 = (C_2 y)^2 = C_2^2 y^2 = y\, y''.$$

Hence the differential equation representing the family is

$$\boxed{yy'' = (y')^2}.$$

Question16

The rate of increase of the population of a city is proportional to the population present at that instant. In the period of 40 years the population increased from 30,000 to 40,000 . At any time t the population is $(a)(b)^{\frac{t}{40}}$. Then the value of a and b are respectively MHT CET 2025 (25 Apr Shift 2)

Options:

A. $30,000, \frac{2}{3}$

B. $30,000, \frac{4}{3}$

C. $40,000, \frac{2}{3}$

D. $40,000, \frac{3}{4}$

Answer: B

Solution:

Answer: $a = 30{,}000,\ b = \dfrac{4}{3}$

Model: $P(t) = a\, b^{t/40}$.

- At $t = 0$: $P(0) = a = 30{,}000 \Rightarrow a = 30{,}000$.
- At $t = 40$: $P(40) = a\, b = 40{,}000 \Rightarrow 30{,}000 \cdot b = 40{,}000 \Rightarrow b = \dfrac{40{,}000}{30{,}000} = \dfrac{4}{3}$.

So the pair is **(30,000, 4/3)**.

Question17

The equation of the curve passing through origin and satisfying $\left(1+x^2\right) \frac{\mathrm{dy}}{\mathrm{dx}} + 2xy = 4x^2$ is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $y\left(1+x^2\right) = 4x^3$

B. $4\left(1+x^{2}\right)=4+y^{2}$

C. 3y $\left(1+x^{2}\right)=4x^{3}$

D. $1+\mathrm{y}^{2}=4x^{3}+1$

Answer: C

Solution:

Answer: $\boxed{3y(1+x^2)=4x^3}$ (Option C)

Solve

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2.$$

Divide by $1+x^2$:

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}.$$

Linear DE. Integrating factor

$$\mu=\exp\int\frac{2x}{1+x^2}\,dx=\exp\left(\ln(1+x^2)\right)=1+x^2.$$

Then

$$\frac{d}{dx}\left[(1+x^2)y\right]=(1+x^2)\cdot\frac{4x^2}{1+x^2}=4x^2.$$

Integrate:

$$(1+x^2)y=\int 4x^2\,dx=\frac{4}{3}x^3+C.$$

Through the origin $(0,0)\Rightarrow C=0$. Hence

$$3y(1+x^2)=4x^3.$$

Question18

The order of the differential equation whose general solution is given by $\mathrm{y}=(\mathrm{C}_1+\mathrm{C}_2)\sin(\mathrm{x}+\mathrm{C}_3)-\mathrm{C}_4\mathrm{e}^{x+\mathrm{C}_5}$ is (where $\mathrm{C}_1,\mathrm{C}_2,\mathrm{C}_3,\mathrm{C}_4,\mathrm{C}_5$ are arbitrary constants) MHT CET 2025 (25 Apr Shift 2)

Options:

A. 5

B. 4

C. 2

D. 3

Answer: D

Solution:

Reason:

Simplify the given family:

$$y = (C_1 + C_2)\sin(x + C_3) - C_4 e^{x+C_5} = A\sin(x+\phi) - Be^x,$$

where $A = C_1 + C_2$, $\phi = C_3$, and $B = C_4 e^{C_5}$.

Thus there are **three independent arbitrary constants** (A, ϕ, B).

A general solution with n independent constants corresponds to an n-th order differential equation, so the order is **3**.

Question19

The general solution of differential equation $(y^2 - x^2)\,\mathrm{d}x = xy\,\mathrm{dy}\ (x \neq 0)$ is MHT CET 2025 (25 Apr Shift 2)

Options:

A. $2x^2 \log x + \mathrm{y}^2 + 2\mathrm{c}x^2 = 0$, where c is the constant of integration

B. $2x^2 \log x - \mathrm{y}^2 + 2\mathrm{c}x^2 = 0$, where c is the constant of integration

C. $x^2 \log x + \mathrm{y}^2 + 2\mathrm{c}x^2 = 0$, where c is the constant of integration

D. $x^2 \log x - \mathrm{y}^2 + 2\mathrm{c}x^2 = 0$, where c is the constant of integration

Answer: A

Solution:

$$(y^2 - x^2)\,dx = xy\,dy \quad (x \neq 0)$$

Write y as $y = vx \Rightarrow dy/dx = v + x\,dv/dx$. Then

$$v + x\frac{dv}{dx} = \frac{y^2 - x^2}{xy} = \frac{v^2x^2 - x^2}{vx^2} = \frac{v^2 - 1}{v}.$$

Hence

$$x\frac{dv}{dx} = \frac{v^2-1}{v} - v = -\frac{1}{v} \quad \Rightarrow \quad v\,dv = -\frac{dx}{x}.$$

Integrate:

$$\frac{v^2}{2} = -\ln x + C \Rightarrow v^2 = C - 2\ln x.$$

Since $v = \frac{y}{x}$, we get

$$\frac{y^2}{x^2} = C - 2\ln x \Rightarrow y^2 + 2x^2\ln x - Cx^2 = 0.$$

Let $C = -2c$. Then the general solution is

$$\boxed{2x^2\log x + y^2 + 2cx^2 = 0},$$

Question20

The rate at which the population of a city increases varies as the population. In a period of 20 years, the population increased from 4 lakhs to 6 lakhs. In another 20 years the population will be MHT CET 2025 (25 Apr Shift 1)

Options:

A. 8 lakhs

B. 12 lakhs

C. 9 lakhs

D. 10 lakhs

Answer: C

Solution:

Given:

- Initial population (P_0) at $t = 0$: 4 lakhs
- After $t = 20$ years: $P(20) = 6$ lakhs

Let's find the constant k:

$$6 = 4e^{20k} \implies \frac{3}{2} = e^{20k} \implies \ln\left(\frac{3}{2}\right) = 20k \implies k = \frac{\ln(3/2)}{20}$$

Next, find the population after another 20 years (i.e., at $t = 40$):

$$P(40) = 4e^{40k}$$

But $e^{40k} = (e^{20k})^2 = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$, so

$$P(40) = 4 \times \frac{9}{4} = 9 \text{ lakhs}$$

So the correct answer is C (9 lakhs),

Question21

The differential equation $x\frac{\mathrm{dy}}{\mathrm{d}x} = 2\mathrm{y}$ represents .... MHT CET 2025 (25 Apr Shift 1)

Options:

A. a family of circles with radius *c*.

B. a family of parabolas with vertex at the origin and axis along the positive Y-axis

C. a family of parabolas with vertex the at origin and axis along the positive X-axis

D. a family of ellipses

Answer: B

Solution:

Differential equation $x\frac{dy}{dx}=2y$:
$\frac{dy}{y}=\frac{2\,dx}{x}\Rightarrow \ln y=2\ln x+C\Rightarrow y=Cx^2$.
This is a family $y=ax^2$ → parabolas with vertex at the origin, axis along +Y.

Answer: B) **a family of parabolas with vertex at the origin and axis along the positive Y-axis.**

Question22

The solution of the equation $x^2\mathrm{y}-x^3\frac{\mathrm{dy}}{\mathrm{d}x}=\mathrm{y}^4\cos x$, where $\mathrm{y}(0)=1$, is MHT CET 2025 (25 Apr Shift 1)

Options:

A. $\mathrm{y}^3=3x^2\sin x$

B. $x^3=3y^3\sin x$

C. $x^3=y^3\sin x$

D. $\mathrm{y}^3=4x^3\sin x$

Answer: B

Solution:

Answer: B — $x^3=3y^3\sin x$

Solve $x^2y-x^3\frac{dy}{dx}=y^4\cos x$, $y(0)=1$.

Divide by y^4 and set $u=y^{-3}$ so $u'=-3y^{-4}y'$:

$$-\frac{x^3y'}{y^4}+\frac{x^2}{y^3}-\cos x=0\ \Rightarrow\ \frac{x^3}{3}u'+x^2u=\cos x.$$

Linear DE:

$$u'+\frac{3}{x}u=\frac{3\cos x}{x^3}.$$

IF $=\exp\int\frac{3}{x}dx=x^3$. Then

$$\frac{d}{dx}(x^3u)=3\cos x\ \Rightarrow\ x^3u=3\sin x+C.$$

Since $u=y^{-3}$ and $y(0)=1$ gives $C=0$:

$$\frac{x^3}{y^3}=3\sin x\ \Rightarrow\ \boxed{x^3=3y^3\sin x}.$$

Question23

$\mathrm{y}=\mathrm{e}^x(\mathrm{A}\cos x+\mathrm{B}\sin x)$ **is the solution of the differential equation MHT CET 2025 (25 Apr Shift 1)**

Options:

A. $x^2 \frac{d^2y}{dx^2} + (1 + y^2) = 0$

B. $\frac{d^2y}{dx^2} - \frac{dy}{dx} + y = 0$

C. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

D. $x\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$

Answer: C

Solution:

1. The General Solution

$$y = e^x(A\cos x + B\sin x)$$

2. First Derivative ($\frac{dy}{dx}$)

Use the product rule: $(uv)' = u'v + uv'$, where $u = e^x$ and $v = A\cos x + B\sin x$.

$$\frac{dy}{dx} = (e^x)(A\cos x + B\sin x) + (e^x)(-A\sin x + B\cos x)$$

Notice that the first term is exactly y:

$$\frac{dy}{dx} = y + e^x(-A\sin x + B\cos x)$$

Let's call the term with the constants y' for simplicity in the next step.

$$\frac{dy}{dx} = y + y_1$$

where $y_1 = e^x(-A\sin x + B\cos x)$.

3. Second Derivative ($\frac{d^2y}{dx^2}$)

Now, take the derivative of $\frac{dy}{dx}$:

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{d}{dx}[e^x(-A\sin x + B\cos x)]$$

Apply the product rule again to the second term:

$$\frac{d}{dx}[e^x(-A\sin x + B\cos x)] = (e^x)(-A\sin x + B\cos x) + (e^x)(-A\cos x - B\sin x)$$

Notice the terms:

- $e^x(-A\sin x + B\cos x)$ is y_1.
- $e^x(-A\cos x - B\sin x) = -e^x(A\cos x + B\sin x) = -y$.

Substitute these back:

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + y_1 + (-y)$$

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + y_1 - y$$

4. Eliminating the Remaining Term (y_1)

From the first derivative step, we have:

$$y_1 = \frac{dy}{dx} - y$$

Substitute this back into the second derivative equation:

$$\frac{d^2y}{dx^2} = \frac{dy}{dx} + \left(\frac{dy}{dx} - y\right) - y$$

$$\frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 2y$$

5. Final Differential Equation

Rearranging the terms to match the options:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

This matches **Option C**.

The correct differential equation is $\mathbf{\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0}$.

Question24

Solution of $(2y - x)\frac{dy}{dx} = 1$ is MHT CET 2025 (23 Apr Shift 2)

Options:

A. $x = 2(y-1) + ce^{-y}$, where c is the constant of integration

B. $x = 2(y-1) + ce^{-x}$, where c is the constant of integration

C. $y = 2(x-1) + ce^{-x}$, where c is the constant of integration

D. $y = 2(x-1) + ce^{-y}$, where c is the constant of integration

Answer: A

Solution:

Answer: $\boxed{x = 2(y-1) + Ce^{-y}}$ **(option A)**

Solve

$$(2y - x)\frac{dy}{dx} = 1 \quad \Rightarrow \quad \frac{dx}{dy} = 2y - x.$$

This is a linear differential equation in $x(y)$:

$$\frac{dx}{dy} + x = 2y.$$

Integrating factor $= \exp\int 1\,dy = e^{y}$.

Multiply through:

$$\frac{d}{dy}(xe^{y}) = 2y\,e^{y}.$$

Integrate:

$$xe^{y} = 2\int ye^{y}\,dy = 2\,(y-1)e^{y} + C.$$

Divide by e^{y}:

$$x = 2(y-1) + Ce^{-y}.$$

That's the general solution.

Question25

The integrating factor of $\mathrm{y} + \frac{\mathrm{d}}{\mathrm{d}x}(x\mathrm{y}) = x(\sin x + \log x)$ **is MHT CET 2025 (23 Apr Shift 2)**

Options:

A. x

B. $\log x^2$

C. x^2

D. x^3

Answer: C

Solution:

Answer: $\boxed{x^2}$

Given:

$$y+\frac{d}{dx}(xy)=x(\sin x+\log x).$$

Since $\frac{d}{dx}(xy)=x\frac{dy}{dx}+y$, the equation becomes

$$x\frac{dy}{dx}+2y=x(\sin x+\log x).$$

Divide by x:

$$\frac{dy}{dx}+\frac{2}{x}y=\sin x+\log x,$$

which is linear in y with $P(x)=\frac{2}{x}$.

Integrating factor:

$$\mu(x)=e^{\int P(x)\,dx}=e^{\int \frac{2}{x}\,dx}=e^{2\ln x}=x^2.$$

So the integrating factor is x^2 (option C).

Question26

The differential equation whose solution is $\mathrm{A}x^2+\mathrm{B}y^2=1$ where A and B are arbitrary constants is of MHT CET 2025 (23 Apr Shift 2)

Options:

A. degree 1 and order 2

B. degree 2 and order 1

C. degree 3 and order 2

D. degree 1 and order 3

Answer: A

Solution:

Answer: degree 1 and order 2 (Option A)

From $Ax^2 + By^2 = 1$ (with parameters A, B), eliminate the constants by differentiating:

1. Differentiate:

$$2Ax + 2Byy' = 0 \Rightarrow Ax + Byy' = 0.$$

2. Differentiate again:

$$A + B\left((y')^2 + yy''\right) = 0.$$

Eliminate A, B using the first in the second:

$$A = -\frac{Byy'}{x} \Rightarrow -\frac{Byy'}{x} + B\left((y')^2 + yy''\right) = 0.$$

Divide by $B \neq 0$:

$$(y')^2 + yy'' - \frac{yy'}{x} = 0.$$

This differential equation is **second order** (highest derivative y'') and **first degree** (power of y'' is 1).

Question27

The rate at which a substance cools in moving air, is proportional to the difference between the temperature of the substance and that of air. The temperature of air is 290 K and the substance cools from 370 K to 330 K in 10 minutes. Then the time to cool the substance upto 295 K is MHT CET 2025 (23 Apr Shift 1)

Options:

A. 40 min

B. 96 min

C. 50 min

D. 60 min

Answer: A

Solution:

Formula

Newton's Law of Cooling:

$$T(t) = T_{\text{air}} + (T_0 - T_{\text{air}})e^{-kt}$$

where:

- $T(t)$ is the temperature after time t
- T_{air} is the ambient air temperature (290 K)
- T_0 is the initial temperature of the substance
- k is a constant
- t is time elapsed

Using Given Values

1. Initial: $T_0 = 370 \text{ K}$
2. At $t_1 = 10 \text{ min}, T_1 = 330 \text{ K}$

Plug into the formula:

$$330 = 290 + (370 - 290)e^{-10k} \implies 40 = 80e^{-10k} \implies \frac{1}{2} = e^{-10k} \implies -10k = \ln\left(\frac{1}{2}\right)$$

Time to Cool to 295 K

$$295 = 290 + 80e^{-kt} \implies 5 = 80e^{-kt} \implies e^{-kt} = \frac{1}{16} \implies -kt = \ln\left(\frac{1}{16}\right) = -\ln 16 = -$$

Using $k = \frac{\ln 2}{10}$:

$$-\frac{\ln 2}{10}t = -4\ln 2 \implies t = 40 \text{ min}$$

Final Answer

The time required to cool the substance from 370 K to 295 K is **40 minutes** (Option A).

Question28

If $x\frac{\text{dy}}{\text{d}x} = \text{y}(\log \text{y} - \log x + 1)$, then the solution of the equation is MHT CET 2025 (23 Apr Shift 1)

Options:

A. $\log \frac{x}{\text{y}} = \text{cy}$, where c is the constant of integration

B. $\log \frac{\text{y}}{x} = \text{cy}$, where c is the constant of integration

C. $\log \frac{x}{\text{y}} = \text{c}x$, where c is the constant of integration

D. $\log \frac{\text{y}}{x} = \text{c}x$, where c is the constant of integration

Answer: D

Solution:

Solve

$$x\frac{dy}{dx} = y(\log y - \log x + 1) = y\left(\log \frac{y}{x} + 1\right).$$

Let $u = \log\left(\frac{y}{x}\right)$. Then $y = xe^u$ and

$$\frac{dy}{dx} = e^u + xe^u u' = e^u(1 + xu').$$

Plug in:

$$xe^u(1 + xu') = xe^u(u + 1) \Rightarrow 1 + xu' = u + 1 \Rightarrow xu' = u.$$

So

$$\frac{du}{u} = \frac{dx}{x} \Rightarrow \ln|u| = \ln x + C \Rightarrow u = Cx.$$

Therefore,

$$\boxed{\log \frac{y}{x} = c\,x}.$$

Question29

The order and degree of the differential equation $\sqrt{\frac{\mathrm{dy}}{\mathrm{d}x}} - 4\frac{\mathrm{dy}}{\mathrm{d}x} - 7x = 0$ is respectively MHT CET 2025 (23 Apr Shift 1)

Options:

A. 1,2

B. 2,1

C. 2,2

D. 3,1

Answer: A

Solution:

Answer: 1, 2

- Highest derivative present is $y' = \frac{dy}{dx}$ ⇒ **order = 1.**
- Remove the radical to make it polynomial in y':

$$\sqrt{y'} - 4y' - 7x = 0 \Rightarrow \sqrt{y'} = 4y' + 7x$$

Squaring,

$$y' = (4y' + 7x)^2,$$

which is a polynomial in y' whose highest power is $(y')^2$ ⇒ **degree = 2.**

Question30

The differential equation of all circles touching the Y-axis at the origin and centre on the X -axis is MHT CET 2025 (23 Apr Shift 1)

Options:

A. $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

B. $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

C. $2x^2 + y^2 + xy\frac{dy}{dx} = 0$

D. $x^2 - 2y^2 + 2xy\frac{dy}{dx} = 0$

Answer: B

Solution:

Answer: $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$ **(Option B)**

Let the circle have center $(a, 0)$ on the x-axis and be tangent to the y-axis at the origin.
Then its radius equals $|a|$ and the family is:

$$(x-a)^2 + y^2 = a^2 \implies x^2 + y^2 - 2ax = 0. \qquad (1)$$

Differentiate (1) w.r.t. x:

$$2x + 2y\,y' - 2a = 0. \qquad (2)$$

From (1), $2a = \frac{x^2+y^2}{x}$. Substitute into (2):

$$2x + 2y\,y' - \frac{x^2+y^2}{x} = 0 \Rightarrow x^2 - y^2 + 2xy\,y' = 0.$$

So the required differential equation is

$$\boxed{x^2 - y^2 + 2xy\frac{dy}{dx} = 0}.$$

Question31

The differential equation satisfied by $y = X\sin(6t+5) + Y\cos(6t+5)$ is (where X and Y are constants) MHT CET 2025 (22 Apr Shift 2)

Options:

A. $\frac{d^2y}{dt^2} + 6y = 0$

B. $\frac{d^2y}{dt^2} = 0$

C. $\frac{d^2y}{dt^2} + 36y = 0$

D. $\frac{d^2y}{dt^2} + 25y = 0$

Answer: C

Solution:

1. The General Solution

$$y = X\sin(6t+5) + Y\cos(6t+5)$$

2. First Derivative $\left(\frac{dy}{dt}\right)$

Using the chain rule, $\frac{d}{dt}\sin(at+b) = a\cos(at+b)$ and $\frac{d}{dt}\cos(at+b) = -a\sin(at+b)$:

$$\frac{dy}{dt} = X\cdot 6\cos(6t+5) + Y\cdot[-6\sin(6t+5)]$$

$$\frac{dy}{dt} = 6X\cos(6t+5) - 6Y\sin(6t+5)$$

3. Second Derivative $\left(\frac{d^2y}{dt^2}\right)$

$$\frac{d^2y}{dt^2} = 6X\cdot[-6\sin(6t+5)] - 6Y\cdot[6\cos(6t+5)]$$

$$\frac{d^2y}{dt^2} = -36X\sin(6t+5) - 36Y\cos(6t+5)$$

4. Eliminate Constants

Factor out -36 from the second derivative:

$$\frac{d^2y}{dt^2} = -36\left[X\sin(6t+5) + Y\cos(6t+5)\right]$$

Substitute the original function y back into the equation:

$$\frac{d^2y}{dt^2} = -36y$$

5. Final Differential Equation

Rearrange the equation into the standard homogeneous form:

$$\frac{d^2y}{dt^2} + 36y = 0$$

Question32

A wet substance in the open air loses its moisture at a rate proportional to the moisture content. If a sheet, hung in the open air, loses half its moisture during the first hour, then 90% of the moisture will be lost in ...... hours. MHT CET 2025 (22 Apr Shift 2)

Options:

A. $2\log_2 10$

B. $\frac{4\log 10}{\log 2}$

C. $\log_2 10$

D. $\frac{3 \log 10}{\log 2}$

Answer: C

Solution:

Answer: $\log_2 10$ **hours.**

Moisture decays exponentially: $M(t) = M_0 \cdot r^t$.
Given it halves in 1 hour $\Rightarrow r = \frac{1}{2}$.

For 90% lost, 10% remains:

$$\left(\frac{1}{2}\right)^t = 0.1 = \frac{1}{10} \Rightarrow 2^{-t} = 10^{-1} \Rightarrow 2^t = 10 \Rightarrow t = \log_2 10.$$

Question33

The general solution of the differential equation. $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ **is MHT CET 2025 (22 Apr Shift 2)**

Options:

A. $\log \tan\left(\frac{y}{2}\right) = c - 2\sin\frac{x}{2}$, where c is the constant of integration

B. $\log \tan\left(\frac{y}{4}\right) = c - 2\sin\left(\frac{x}{2}\right)$, where c is the constant of integration

C. $\log\left[\tan\left(\frac{y}{2} + \frac{\pi}{4}\right)\right] = c - 2\sin x$, where c is the constant of integration

D. $\log\left[\tan\left(\frac{y}{4} + \frac{\pi}{4}\right)\right] = c - 2\sin\frac{x}{2}$, where c is the constant of integration

Answer: B

Solution:

Answer: B — $\log \tan \frac{y}{4} = c - 2 \sin \frac{x}{2}$

Solve

$$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right).$$

Move the sine term:

$$\frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right).$$

Use $\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2}$ with
$A = \frac{x-y}{2}$, $B = \frac{x+y}{2}$:

$$\frac{dy}{dx} = 2\cos\left(\frac{x}{2}\right)\sin\left(-\frac{y}{2}\right) = -2\cos\left(\frac{x}{2}\right)\sin\left(\frac{y}{2}\right).$$

Separate:

$$\frac{dy}{\sin(y/2)} = -2\cos\left(\frac{x}{2}\right)dx.$$

Let $u = \frac{y}{2} \Rightarrow dy = 2\,du$:

$$2\int \csc u\,du = -2\int \cos\left(\frac{x}{2}\right)dx.$$

Recall $\int \csc u\,du = \ln\left|\tan\frac{u}{2}\right|$:

$$2\ln\left|\tan\frac{u}{2}\right| = -2\cdot 2\sin\left(\frac{x}{2}\right) + C$$

$$\Rightarrow \ln\left|\tan\frac{y}{4}\right| = -2\sin\left(\frac{x}{2}\right) + C.$$

So the general solution is

$$\boxed{\log\tan\frac{y}{4} = c - 2\sin\frac{x}{2}}.$$

Question34

The equation of the curve passing through $\left(2, \frac{9}{2}\right)$ and having the slope $\left(1 - \frac{1}{x^2}\right)$ at (x, y) is MHT CET 2025 (22 Apr Shift 2)

Options:

A. $xy = x^2 + 2x + 1$

B. $xy = x^2 + x + 2$

C. $xy = x^2 + x + 5$

D. $xy = x^2 + 2x + 5$

Answer: A

Solution:

Given slope:

$$\frac{dy}{dx} = 1 - \frac{1}{x^2}.$$

Integrate:

$$y = \int \left(1 - \frac{1}{x^2}\right) dx = x + \frac{1}{x} + C.$$

Use the point $\left(2, \frac{9}{2}\right)$:

$$\frac{9}{2} = 2 + \frac{1}{2} + C \Rightarrow C = 2.$$

So

$$y = x + \frac{1}{x} + 2.$$

Multiply by x:

$$xy = x^2 + 2x + 1.$$

$$\boxed{xy = x^2 + 2x + 1} \quad \text{(Option A)}$$

Question35

If $\mathrm{y} = \mathrm{y}(x)$ and $\left(\frac{2+\sin x}{\mathrm{y}+1}\right) \frac{\mathrm{dy}}{\mathrm{d}x} = -\cos x$, $y(0) = 1$, then $y\left(\frac{\pi}{2}\right) =$ MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $-\frac{1}{3}$

D. 1

Answer: A

Solution:

$$\frac{2+\sin x}{y+1}\frac{dy}{dx} = -\cos x, \qquad y(0) = 1$$

Separate:

$$\frac{1}{y+1}\frac{dy}{dx} = -\frac{\cos x}{2+\sin x} \Rightarrow \frac{d}{dx}\ln(y+1) = -\frac{\cos x}{2+\sin x}.$$

Let $u = 2+\sin x \Rightarrow du = \cos x\, dx$:

$$\ln(y+1) = -\ln(2+\sin x) + C \;\Rightarrow\; (y+1)(2+\sin x) = C'.$$

Use $x = 0,\ y = 1$: $(1+1)(2+0) = 4 \Rightarrow C' = 4.$

So

$$y+1 = \frac{4}{2+\sin x} \;\Rightarrow\; y = \frac{2-\sin x}{2+\sin x}.$$

At $x = \frac{\pi}{2}$ $(\sin x = 1)$:

$$y\left(\frac{\pi}{2}\right) = \frac{2-1}{2+1} = \boxed{\frac{1}{3}}.$$

Question36

The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2\log\left(\frac{d^2y}{dx^2}\right)$ is MHT CET 2025 (22 Apr Shift 1)

Options:

A. 1

B. 2

C. 3

D. Not defined

Answer: D

Solution:

The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^3y}{dx^3}\right)$ is **Not defined**.

Explanation

Order and Degree Defined

1. **Order:** The order of a differential equation is the order of the highest derivative present. In this equation, the highest derivative is $\frac{d^3y}{dx^3}$, so the **Order is 3**.
2. **Degree:** The degree of a differential equation is the highest power (exponent) of the highest-order derivative **after the equation has been expressed as a polynomial in its derivatives.**

Determining the Degree

The given equation contains the term $x^2 \log\left(\frac{d^3y}{dx^3}\right)$.

A differential equation must be a **polynomial in all its derivatives** (e.g., $\left(\frac{d^3y}{dx^3}\right)^1$, $\left(\frac{d^3y}{dx^3}\right)^2$, etc.) for its degree to be defined.

Since the highest-order derivative, $\frac{d^3y}{dx^3}$, appears inside a **non-polynomial function** ($\log$), the equation cannot be written as a polynomial in its derivatives.

Therefore, the **degree of the differential equation is Not defined**.

Question37

If $\mathrm{y} + \frac{\mathrm{d}}{\mathrm{d}x}(x\mathrm{y}) = x(\sin x + \log x)$ then MHT CET 2025 (22 Apr Shift 1)

Options:

A. $\mathrm{y} = \cos x + \frac{2\sin x}{x} + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{\mathrm{c}}{x^2}$,where c is the constant of integration.

B. $\mathrm{y} = -\cos x - \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{\mathrm{c}}{x^2}$where c is the constant of integration.

C. $\mathrm{y} = -\cos x + \frac{2}{x}\sin x + \frac{2}{x^2}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{\mathrm{c}}{x^2}$,where c is the constant of integration.

D. $\mathrm{y} = \cos x - \frac{2}{x}\sin x + \frac{2}{x^3}\cos x + \frac{x}{3}\log x - \frac{x}{9} + \frac{\mathrm{c}}{x^2}$,where c is the constant of integration.

Answer: C

Solution:

Answer: C

Start with

$$y+\frac{d}{dx}(xy)=x(\sin x+\log x)$$

Since $\frac{d}{dx}(xy)=x\,y'+y$,

$$x\,y'+2y=x(\sin x+\log x)\quad\Rightarrow\quad y'+\frac{2}{x}y=\sin x+\log x.$$

This is linear with integrating factor $\mu=e^{\int(2/x)\,dx}=x^2$. Hence

$$(x^2y)'=x^2(\sin x+\log x)=\underbrace{\int x^2\sin x\,dx}_{I_1}+\underbrace{\int x^2\log x\,dx}_{I_2}+C.$$

Compute:

$$I_1=\int x^2\sin x\,dx=-x^2\cos x+2x\sin x+2\cos x,$$

$$I_2=\int x^2\log x\,dx=\frac{x^3}{3}\log x-\frac{x^3}{9}.$$

So

$$x^2y=-x^2\cos x+2x\sin x+2\cos x+\frac{x^3}{3}\log x-\frac{x^3}{9}+C,$$

and dividing by x^2,

$$\boxed{y=-\cos x+\frac{2}{x}\sin x+\frac{2}{x^2}\cos x+\frac{x}{3}\log x-\frac{x}{9}+\frac{C}{x^2}}$$

Question38

The population of a town increases at a rate proportional to the population at that time. If the population increases from forty thousand to eighty thousand in 20 years, then the population in another 40 years will be MHT CET 2025 (22 Apr Shift 1)

Options:

A. 240000

B. 160000

C. 320000

D. 640000

Answer: C

Solution:

320,000 (Option C)

Exponential growth ⇒ population multiplies by a constant each equal time.

Given: $40{,}000 \rightarrow 80{,}000$ in 20 years ⇒ it **doubles every 20 years.**

Another 40 years = two more 20-year periods ⇒ multiply by $2^2 = 4$:

- From 80,000: $80{,}000 \times 4 = 320{,}000.$
- Equivalently from the start: $40{,}000 \times 2^{60/20} = 40{,}000 \times 8 = 320{,}000.$

Question39

A particular solution of $3\mathrm{e}^x \tan \mathrm{y} \mathrm{d}x + (1 - \mathrm{e}^x) \sec^2 \mathrm{y} \mathrm{dy} = 0$ with $y(1) = \frac{\pi}{4}$ is MHT CET 2025 (21 Apr Shift 2)

Options:

A. $\tan \mathrm{y} = \left(\frac{1-\mathrm{e}^3}{1-\mathrm{e}^x}\right)^3$

B. $\tan \mathrm{y} = \left(\frac{1-\mathrm{e}^2}{1-\mathrm{e}^x}\right)^3$

C. $\tan \mathrm{y} = \left(\frac{1-\mathrm{e}}{1-\mathrm{e}^x}\right)^3$

D. $\tan \mathrm{y} = \left(\frac{1-\mathrm{e}^x}{1-\mathrm{e}}\right)^3$

Answer: D

Solution:

Answer: D

Given

$$3e^x \tan y\, dx + (1 - e^x) \sec^2 y\, dy = 0, \qquad y(1) = \frac{\pi}{4}.$$

Rewrite as

$$\frac{dy}{dx} = -\frac{3e^x}{1 - e^x} \cdot \frac{\tan y}{\sec^2 y} = -\frac{3e^x}{1 - e^x} \sin y \cos y.$$

Separate:

$$\frac{dy}{\sin y \cos y} = -\frac{3e^x}{1 - e^x}\, dx.$$

Note $\int \frac{dy}{\sin y \cos y} = \ln|\tan y|$.
For the right side, let $u = 1 - e^x \Rightarrow du = -e^x dx$:

$$\int -\frac{3e^x}{1 - e^x}\, dx = \int \frac{3\, du}{u} = 3 \ln|u| = 3 \ln|1 - e^x| + C.$$

Thus

$$\ln|\tan y| = 3 \ln|1 - e^x| + C \;\Rightarrow\; \tan y = C'(1 - e^x)^3.$$

Use $y(1) = \pi/4 \Rightarrow \tan y = 1$:

$$1 = C'(1 - e)^3 \;\Rightarrow\; C' = \frac{1}{(1 - e)^3}.$$

Hence

$$\boxed{\tan y = \left(\frac{1 - e^x}{1 - e}\right)^3}.$$

Question40

The equation of the curve passing through the origin and satisfying the equation $\left(1 + x^2\right) \frac{\mathrm{dy}}{\mathrm{dx}} + 2xy = 4x^2$, is MHT CET 2025 (21 Apr Shift 2)

Options:

A. $3\left(1 + x^2\right) \mathrm{y} = 4x^3$

B. $3\left(1 - x^2\right) \mathrm{y} = 4x^3$

C. $3\left(1 + x^2\right) = x^3$

D. $4\left(1 - x^2\right) = x^3$

Answer: A

Solution:

$3(1+x^2)\,y=4x^3$

Work:

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2 \Rightarrow y'+\frac{2x}{1+x^2}y=\frac{4x^2}{1+x^2}.$$

Linear; IF $=\exp\int\frac{2x}{1+x^2}dx=1+x^2$.

$$\frac{d}{dx}\left[(1+x^2)y\right]=4x^2 \Rightarrow (1+x^2)y=\frac{4}{3}x^3+C.$$

Through origin $\Rightarrow C=0$. Multiply by 3:

$$\boxed{3(1+x^2)\,y=4x^3}.$$

Question41

The differential equation of all circles having their centres on the line $y=5$ and touching X -axis is ...... MHT CET 2025 (21 Apr Shift 2)

Options:

A. $(5-y)\frac{dy}{dx}+y^2-10y=0$

B. $(5-y)^2\frac{d^2y}{dx^2}+y^2-10y=0$

C. $(5-y)\frac{dy}{dx}+y-10=0$

D. $(5-y)^2\left(\frac{dy}{dx}\right)^2+y^2-10y=0$

Answer: D

Solution:

Answer: D — $(5-y)^2\left(\frac{dy}{dx}\right)^2+y^2-10y=0$

Let the circle have center $(h,5)$ (since centers lie on $y=5$) and it touches the x-axis, so radius $=5$.
Family:

$$(x-h)^2+(y-5)^2=25. \qquad (1)$$

Differentiate (1) w.r.t. x:

$$2(x-h)\cdot 1+2(y-5)y'=0 \Rightarrow h=x+(y-5)y'. \qquad (2)$$

Eliminate h:

$$(x-h)^2=\left(x-(x+(y-5)y')\right)^2=\left(-(y-5)y'\right)^2=(y-5)^2(y')^2.$$

Plug in (1):

$$(y-5)^2(y')^2+(y-5)^2=25 \Rightarrow (5-y)^2\left(\frac{dy}{dx}\right)^2+(5-y)^2-25=0.$$

Since $(5-y)^2-25=y^2-10y$, we get

$$\boxed{(5-y)^2\left(\frac{dy}{dx}\right)^2+y^2-10y=0}.$$

Question42

In a culture bacteria count is $1,00,000$ initially. The number increases by 10% in first 2 hours. In how many hours will the count reach $2,00,000$, if the rate of growth of bacteria is proportional to the number present? MHT CET 2025 (21 Apr Shift 2)

Options:

A. $\frac{2\log\left(\frac{11}{10}\right)}{\log 2}$

B. $\frac{\log\left(\frac{11}{10}\right)}{\log 2}$

C. $\frac{2\log 2}{\log\left(\frac{11}{10}\right)}$

D. $\frac{\log(2)}{\log\left(\frac{11}{10}\right)}$

Answer: C

Solution:

$\frac{2 \log 2}{\log(11/10)}$ hours (Option C)

Reason: Exponential growth $\Rightarrow N(t) = N_0 \cdot a^t$.

Given 10% increase in 2 hours: multiplier over 2 hours is $1.1 \Rightarrow$ per t hours, factor is $(1.1)^{t/2}$.

Set $N(t) = 2N_0$:

$$(1.1)^{t/2} = 2 \Rightarrow \frac{t}{2} = \frac{\log 2}{\log(1.1)} \Rightarrow t = \frac{2 \log 2}{\log(11/10)}.$$

Question43

A particular solution of $\frac{dy}{dx} = (x + 9y)^2$, when $x = 0, y = \frac{1}{27}$ is MHT CET 2025 (21 Apr Shift 1)

Options:

A. $3x + 27y = \tan\left[3\left(x + \frac{\pi}{12}\right)\right]$

B. $3x + 27y = \tan\left(x + \frac{\pi}{4}\right)$

C. $3x + 27y = \tan\left(x + \frac{\pi}{12}\right)$

D. $3x + 27y = \tan\left[3\left(x + \frac{\pi}{4}\right)\right]$

Answer: A

Solution:

1. Substitution

Let $v = x + 9y$. Differentiate v with respect to x:

$$\frac{dv}{dx} = \frac{d}{dx}(x + 9y)$$

$$\frac{dv}{dx} = 1 + 9\frac{dy}{dx}$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{1}{9}\left(\frac{dv}{dx} - 1\right)$$

Substitute v and $\frac{dy}{dx}$ into the original equation:

$$\frac{1}{9}\left(\frac{dv}{dx} - 1\right) = v^2$$

$$\frac{dv}{dx} - 1 = 9v^2$$

$$\frac{dv}{dx} = 9v^2 + 1$$

2. Solve the Separable Equation

The new equation is separable.

$$\frac{dv}{9v^2 + 1} = dx$$

Integrate both sides:

$$\int \frac{1}{9v^2 + 1} dv = \int dx$$

Use the standard integral formula: $\int \frac{1}{a^2u^2+1}du = \frac{1}{a}\tan^{-1}(au)$. We have $\int \frac{1}{(3v)^2+1}dv$. Let $u = 3v$, so $du = 3dv$, or $dv = \frac{1}{3}du$.

$$\int \frac{1}{u^2+1} \cdot \frac{1}{3}du = \frac{1}{3}\tan^{-1}(u) = \frac{1}{3}\tan^{-1}(3v)$$

So, integrating both sides gives:

$$\frac{1}{3}\tan^{-1}(3v) = x + C$$

3. General Solution

Substitute back $v = x + 9y$:

$$\frac{1}{3}\tan^{-1}(3(x+9y)) = x + C$$

$$\tan^{-1}(3x+27y) = 3x + 3C$$

Let $K = 3C$ be the new arbitrary constant.

$$\tan^{-1}(3x+27y) = 3x + K$$

$$3x + 27y = \tan(3x + K)$$

4. Apply Initial Condition

Use the condition $x = 0, y = \frac{1}{27}$ to find K:

$$3(0) + 27\left(\frac{1}{27}\right) = \tan(3(0) + K)$$

$$0 + 1 = \tan(K)$$

$$K = \tan^{-1}(1)$$

$$K = \frac{\pi}{4}$$

(Using the principal value)

5. Particular Solution

Substitute $K = \frac{\pi}{4}$ back into the general solution:

$$3x + 27y = \tan\left(3x + \frac{\pi}{4}\right)$$

This solution is in the form of Option A: $3x + 27y = \tan\left[3\left(x + \frac{\pi}{12}\right)\right]$, if we factor out 3 from the argument of the tangent function:

$$\tan\left(3x + \frac{\pi}{4}\right) = \tan\left(3\left(x + \frac{\pi/4}{3}\right)\right) = \tan\left(3\left(x + \frac{\pi}{12}\right)\right)$$

The particular solution is:

$$3x + 27y = \tan\left[3\left(x + \frac{\pi}{12}\right)\right]$$

Question44

The general solution of $\frac{dy}{dx} = 2xye^{x^2}$ is MHT CET 2025 (21 Apr Shift 1)

Options:

A. $y = e^{-e^{x^2}} c$, where c is the constant of integration

B. $y = e^{-x^2} c$, where c is the constant of integration

C. $y = e^{e^{x^2}} c$, where c is the constant of integration

D. $y = e^{x^2} c$, where c is the constant of integration

Answer: C

Solution:

$$\frac{dy}{dx} = 2x\, y\, e^{x^2}.$$

1) Recognize it's separable

Bring all y-terms to one side and x-terms to the other:

$$\frac{1}{y}\, dy = 2x\, e^{x^2}\, dx.$$

2) Integrate both sides

Left side:

$$\int \frac{1}{y}\, dy = \ln|y| + C_1.$$

Right side: use the substitution $u = x^2 \Rightarrow du = 2x\, dx$:

$$\int 2x\, e^{x^2}\, dx = \int e^u\, du = e^u + C_2 = e^{x^2} + C_2.$$

Equate:

$$\ln|y| = e^{x^2} + C \quad (\text{where } C = C_2 - C_1).$$

3) Exponentiate and absorb constants

$$|y| = e^{e^{x^2} + C} = e^C\, e^{e^{x^2}}.$$

Let $c = \pm e^C$ (a nonzero constant, and you may also allow $c = 0$ to include the zero solution):

$$\boxed{y = c\, e^{e^{x^2}}}.$$

4) Quick check

If $y = c\, e^{e^{x^2}}$, then

$$\frac{dy}{dx} = c\, e^{e^{x^2}} \cdot e^{x^2} \cdot (2x) = 2x\, e^{x^2}\, y,$$

which matches the differential equation.

Hence the correct option is $y = c\, e^{e^{x^2}}$ (option C).

Question45

The assets of a person reduced in his business such that the rate of reduction is proportional to the square root of the existing assets. If the assets were initially ₹ 10 lakhs and due to loss they reduce to ₹10000 after 3 years, then the number of years required for the person to be bankrupt will be MHT CET 2025 (21 Apr Shift 1)

Options:

A. $\frac{20}{3}$ years

B. $\frac{10}{3}$ years

C. $\frac{10}{9}$ years

D. $\frac{20}{9}$ years

Answer: B

Solution:

Given

- Asset $A(t)$ decreases at a rate proportional to $\sqrt{A}$:

$$\frac{dA}{dt} = -k\sqrt{A}, \quad k > 0$$

- $A(0) = 10\,\text{lakhs} = 1{,}000{,}000$
- $A(3) = 10{,}000$

Solve the ODE

Separate and integrate:

$$\int \frac{dA}{\sqrt{A}} = \int -k\,dt \quad \Rightarrow \quad 2\sqrt{A} = -kt + C$$

Use $A(0) = 10^6 \Rightarrow \sqrt{A} = 1000$:

$$2(1000) = C \Rightarrow C = 2000$$

Use $A(3) = 10{,}000 \Rightarrow \sqrt{A} = 100$:

$$2(100) = -3k + 2000 \Rightarrow 2000 - 3k = 200 \Rightarrow k = 600$$

Time to bankruptcy

Bankruptcy when $A = 0 \Rightarrow \sqrt{A} = 0$:

$$0 = -kt + 2000 \Rightarrow t = \frac{2000}{600} = \frac{10}{3}\text{ years}$$

Answer: B) $\boxed{\frac{10}{3}\text{ years}}$.

Question46

If the differential equation $\frac{dy}{dx}+\frac{x}{y}=\frac{a}{y}$ where a is constant, represents a family of circles then the radius of the circle is ...... MHT CET 2025 (20 Apr Shift 2)

Options:

A. $a+2\mathrm{c}$, where c is the constant of integration

B. $\sqrt{a^2+2c}$, where c is the constant of integration

C. a^2+2c , where c is the constant of integration

D. $\sqrt{a+c}$, where c is the constant of integration

Answer: B

Solution:

Let

$$\frac{dy}{dx}+\frac{x}{y}=\frac{a}{y}.$$

Multiply by y:

$$y\frac{dy}{dx}+x=a.$$

Since $y\frac{dy}{dx}=\frac{d}{dx}\left(\frac{y^2}{2}\right)$,

$$\frac{d}{dx}\left(\frac{y^2}{2}\right)=a-x \quad\Rightarrow\quad \frac{y^2}{2}=ax-\frac{x^2}{2}+C.$$

Thus

$$y^2=2ax-x^2+2C \quad\Rightarrow\quad x^2+y^2-2ax=2C \quad\Rightarrow\quad (x-a)^2+y^2=a^2+2C.$$

This is a circle centered at $(a,0)$ with radius

$$\boxed{R=\sqrt{a^2+2C}}.$$

(With $c=C$ as the constant of integration, this matches option **B.**)

Question47

The particular solution of the differential equation $\cos\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right)=0.5, \mathrm{y}=1$ at $x=0$ is MHT CET 2025 (20 Apr Shift 2)

Options:

A. $\cos\left(\frac{2}{x}\right)=1$

B. $\cos\left(\frac{y}{x-1}\right)=0.5$

C. $\cos\left(\frac{y-1}{x}\right)=0.5$

D. $\cos\left(\frac{y}{x}\right)=0.5$

Answer: C

Solution:

1. Solve the Differential Equation

First, solve the differential equation for $\frac{dy}{dx}$:

$$\cos\left(\frac{dy}{dx}\right) = 0.5$$

The value $0.5 = \frac{1}{2}$ corresponds to the principal value $\frac{\pi}{3}$ for the inverse cosine function.

$$\frac{dy}{dx} = \cos^{-1}(0.5)$$

$$\frac{dy}{dx} = \frac{\pi}{3} + 2n\pi \quad \text{or} \quad -\frac{\pi}{3} + 2n\pi$$

where n is an integer. Since the right side is a constant, we can represent it by a single arbitrary constant, say A.

$$\frac{dy}{dx} = A$$

where A is a constant, and the principal value is $A = \frac{\pi}{3}$.

This is a simple separable equation:

$$dy = A\,dx$$

Integrate both sides to find the general solution:

$$\int dy = \int A\,dx$$

$$y = Ax + C$$

where C is the constant of integration.

2. Apply Initial Condition

Use the condition $y = 1$ at $x = 0$ to find C:

$$1 = A(0) + C$$

$$C = 1$$

The particular solution is:

$$y = Ax + 1$$

$$y - 1 = Ax$$

$$A = \frac{y-1}{x}$$

---

3. Substitute back into the Differential Equation

The constant A represents all possible values of $\frac{dy}{dx}$ from the original equation. We substitute $A = \frac{y-1}{x}$ back into the original differential equation:

$$\cos\left(\frac{dy}{dx}\right) = 0.5$$

$$\cos(A) = 0.5$$

$$\cos\left(\frac{y-1}{x}\right) = 0.5$$

This matches the form of **Option B** and **Option C**.

Wait! Let's check the options carefully:

- B: $\cos\left(\frac{y}{x-1}\right) = 0.5$ (Incorrect, the denominator is $x - 1$)
- C: $\cos\left(\frac{y-1}{x}\right) = 0.5$ (Correct)

The particular solution is $\cos\left(\frac{\mathrm{y}-1}{\mathrm{x}}\right) = \mathbf{0.5}$.

Question48

The solution of $(1 + y^2) + \left(x - e^{\tan^{-1} y}\right) \frac{dy}{dx} = 0$ is MHT CET 2025 (20 Apr Shift 2)

Options:

A. $2xe^{\tan^{-1} y} = e^{2\tan^{-1} y} + k$, where k is the constant of integration

B. $x \cdot e^{\tan^{-1} y} = e^{\tan^{-1} y} + k$, where k is the constant of integration

C. $x \cdot e^{2\tan^{-1} y} = e^{\tan^{-1} y} + k$, where k is the constant of integration

D. $x = 2 + k \cdot e^{-\tan^{-1} y}$, where k is the constant of integration

Answer: A

Solution:

We are given

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$$

or, multiplying by dx,

$$(1+y^2)\,dx+(x-e^{\tan^{-1}y})\,dy=0.$$

Let $M(x,y)=1+y^2$ and $N(x,y)=x-e^{\tan^{-1}y}$.
It is not exact because

$$\frac{\partial M}{\partial y}=2y,\qquad \frac{\partial N}{\partial x}=1.$$

But

$$\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}=1-2y,\qquad \frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\Big/M=\frac{1-2y}{1+y^2}$$

depends only on y.
Hence an integrating factor $\mu(y)$ exists with

$$\frac{\mu'(y)}{\mu(y)}=\frac{1-2y}{1+y^2}\quad\Rightarrow\quad \mu(y)=\exp\big(\arctan y-\ln(1+y^2)\big)=\frac{e^{\arctan y}}{1+y^2}.$$

Multiply the DE by $\mu(y)$:

$$e^{\arctan y}\,dx+\frac{(x-e^{\arctan y})e^{\arctan y}}{1+y^2}\,dy=0.$$

This is exact because

$$\frac{\partial}{\partial y}\left(e^{\arctan y}\right)=\frac{e^{\arctan y}}{1+y^2}=\frac{\partial}{\partial x}\left(\frac{(x-e^{\arctan y})e^{\arctan y}}{1+y^2}\right).$$

Let $\Phi(x,y)$ be a potential with $\Phi_x=e^{\arctan y}$.
Integrate w.r.t. x:

$$\Phi=x\,e^{\arctan y}+g(y).$$

Differentiate w.r.t. y and match $\Phi_y=N\mu$:

$$\Phi_y=\frac{xe^{\arctan y}}{1+y^2}+g'(y)=\frac{(x-e^{\arctan y})e^{\arctan y}}{1+y^2}.$$

Hence

$$g'(y)=-\frac{e^{2\arctan y}}{1+y^2}.$$

Let $\theta=\arctan y\Rightarrow dy=(1+y^2)\,d\theta$. Then

$$g(y)=-\int e^{2\theta}\,d\theta=-\tfrac{1}{2}e^{2\theta}=-\tfrac{1}{2}e^{2\arctan y}.$$

Therefore the implicit solution is

$$\Phi=C\ \Rightarrow\ xe^{\arctan y}-\frac{1}{2}e^{2\arctan y}=C$$

or equivalently

$$\boxed{2x\,e^{\tan^{-1}y}=e^{2\tan^{-1}y}+k}$$

which corresponds to option A.

Question49

The rate of reduction of a persons assets is proportional to the square root of the existing assets. The assets reduced from 25 lakhs to 6.25 lakhs in 2 years. This rate of reduction of his assets will make him bankrupt in MHT CET 2025 (20 Apr Shift 2)

Options:

A. 3 years

B. 5 years

C. 4 years

D. 6 years

Answer: C

Solution:

Let $A(t)$ be the assets (in lakhs).
"Rate of reduction proportional to $\sqrt{A}$" ⇒

$$\frac{dA}{dt} = -k\sqrt{A}, \quad k > 0.$$

Separate and integrate:

$$\int \frac{dA}{\sqrt{A}} = \int -k\,dt \;\Rightarrow\; 2\sqrt{A} = -kt + C.$$

Initial: $A(0) = 25 \Rightarrow \sqrt{A} = 5$, so $C = 10$.

After 2 years: $A(2) = 6.25 \Rightarrow \sqrt{A} = 2.5$:

$$2(2.5) = -2k + 10 \Rightarrow 5 = 10 - 2k \Rightarrow k = 2.5.$$

Bankruptcy when $A = 0 \Rightarrow \sqrt{A} = 0$:

$$0 = -kt + 10 \Rightarrow t = \frac{10}{2.5} = 4 \text{ years}.$$

Answer: C) 4 years.

Question50

The general solution of$x(x-1)\frac{\mathrm{dy}}{\mathrm{d}x} = x^3(2x-1) + (x-2)\mathrm{y}$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. $y(x-1) = x^3 + c(x-1)$, where c is the constant of integration.

B. $\mathrm{y} = x^3(x-1) + \mathrm{c}$, where c is the constant of integration.

C. $y(x-1) = x^3(x-1) + cx^2$, where c is the constant of integration.

D. $y(x-1) = x^3(x-1) + c$, where c is the constant of integration.

Answer: C

Solution:

Solve

$$x(x-1)\frac{dy}{dx} = x^3(2x-1) + (x-2)y.$$

Divide by $x(x-1)$:

$$\frac{dy}{dx} - \frac{x-2}{x(x-1)}y = \frac{x^3}{x} \cdot \frac{2x-1}{x-1} = x^2\frac{2x-1}{x-1}.$$

This is linear $y' + P(x)y = Q(x)$ with

$$P(x) = -\frac{x-2}{x(x-1)} = -\left(\frac{2}{x} - \frac{1}{x-1}\right).$$

Integrating factor:

$$\mu(x) = e^{\int P(x)\,dx} = e^{-\int\left(\frac{2}{x}-\frac{1}{x-1}\right)dx} = e^{-2\ln x + \ln(x-1)} = \frac{x-1}{x^2}.$$

Then

$$\frac{d}{dx}(\mu y) = Q(x)\mu = \left(x^2\frac{2x-1}{x-1}\right)\left(\frac{x-1}{x^2}\right) = 2x-1.$$

Integrate:

$$\mu y = \int (2x-1)\,dx = x^2 - x + C.$$

Thus

$$\frac{x-1}{x^2}y = x^2 - x + C \quad \Rightarrow \quad y(x-1) = x^2(x^2 - x + C) = x^3(x-1) + Cx^2.$$

$$\boxed{y(x-1) = x^3(x-1) + cx^2}$$

Question51

The sum of the degree and order of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[5]{\frac{dy}{dx}} - 5$ is MHT CET 2025 (20 Apr Shift 1)

Options:

A. 1

B. 3

C. 5

D. 7

Answer: D

Solution:

Problem 1: Area Calculation

The area of the region bounded by $y = |x-4|$, $x = 3$, $x = 5$, and the x-axis is requested.

1. Geometric Calculation

The function $y = |x-4|$ is a V-shape with its vertex at $(4, 0)$. The area from $x = 3$ to $x = 5$ is the sum of the areas of two right triangles.

- **Left Triangle (from $x = 3$ to $x = 4$):**
  - Base (b_1): $4 - 3 = 1$
  - Height (h_1): $y(3) = |3-4| = 1$
  - Area $A_1 = \frac{1}{2}b_1h_1 = \frac{1}{2}(1)(1) = \frac{1}{2}$
- **Right Triangle (from $x = 4$ to $x = 5$):**
  - Base (b_2): $5 - 4 = 1$
  - Height (h_2): $y(5) = |5-4| = 1$
  - Area $A_2 = \frac{1}{2}b_2h_2 = \frac{1}{2}(1)(1) = \frac{1}{2}$

$$\text{Total Area} = A_1 + A_2 = \frac{1}{2} + \frac{1}{2} = \mathbf{1}$$

2. Conclusion

The correct mathematical answer is **1**. Since $1 = \frac{17}{17}$, none of the provided options ($\frac{21}{17}, \frac{22}{17}, \frac{23}{17}, \frac{24}{17}$) are correct. The question or options are flawed.

Problem 2: Order and Degree of Differential Equation

The sum of the degree and order of the differential equation $\sqrt{\frac{d^2y}{dx^2}} = \sqrt[5]{\frac{dy}{dx}} - 5$ is requested.

1. Rationalize the Equation

To find the degree, we must first clear all fractional exponents (radicals) by raising both sides to a power that removes the roots. The roots involved are the square root (power $\frac{1}{2}$) and the fifth root (power $\frac{1}{5}$). The least common multiple of the denominators (2 and 5) is 10.

Raise both sides to the power of 10:

$$\left(\sqrt{\frac{d^2y}{dx^2}}\right)^{10} = \left(\sqrt[5]{\frac{dy}{dx}} - 5\right)^{10}$$

$$\left(\frac{d^2y}{dx^2}\right)^{5} = \left(\left(\frac{dy}{dx}\right)^{\frac{1}{5}} - 5\right)^{10}$$

2. Determine Order and Degree

- **Order:** The highest derivative is $\frac{d^2y}{dx^2}$.

$$\text{Order} = \mathbf{2}$$

- **Degree:** The degree is the exponent of the highest-order derivative ($\frac{d^2y}{dx^2}$) after rationalization. In the rationalized equation:

$$\left(\frac{d^2y}{dx^2}\right)^{5} = \left(\left(\frac{dy}{dx}\right)^{\frac{1}{5}} - 5\right)^{10}$$

The highest-order derivative, $\frac{d^2y}{dx^2}$, has a power of **5**.

$$\text{Degree} = \mathbf{5}$$

3. Calculate the Sum

$$\text{Sum} = \text{Order} + \text{Degree} = 2 + 5 = \mathbf{7}$$

Question52

The differential equation whose solution represents the family $x^2y = 4e^x + c$, where c is an arbitrary constant, is MHT CET 2025 (20 Apr Shift 1)

Options:

A. $x\frac{\mathrm{dy}}{\mathrm{d}x} + x\mathrm{y} = 0$

B. $x^2\frac{\mathrm{dy}}{\mathrm{d}x} + (2xy - 4e^x) = 0$

C. $x\frac{\mathrm{dy}}{\mathrm{d}x} + (x-2)\mathrm{y} = 0$

D. $x\frac{\mathrm{dy}}{\mathrm{d}x} + (2-x)\mathrm{y} = 0$

Answer: B

Solution:

1. Differential Equation from a General Solution

The problem asks for the differential equation whose solution is the family $x^2y = 4e^x + c$.

Solution

To find the differential equation, we eliminate the arbitrary constant c by differentiating the solution with respect to x.

1. **Isolate c:**

$$c = x^2y - 4e^x$$

2. **Differentiate implicitly w.r.t. x ($\frac{dc}{dx} = 0$):**

$$\frac{d}{dx}(x^2y) - \frac{d}{dx}(4e^x) = 0$$

3. **Apply the Product Rule** on x^2y:

$$\left(2x \cdot y + x^2 \cdot \frac{dy}{dx}\right) - 4e^x = 0$$

4. **Rearrange:**

$$x^2\frac{dy}{dx} + 2xy - 4e^x = 0$$

$$x^2\frac{dy}{dx} + (2xy - 4e^x) = 0$$

This exactly matches **Option B.**

Question53

The differential equation of all straight lines passing through the point $(1, -1)$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. y $= (x-1)\frac{\text{dy}}{\text{d}x} - 1$

B. $x = (x-1)\frac{\text{dy}}{\text{d}x} + 1$

C. y $= (x-1)\frac{\text{dy}}{\text{d}x}$

D. y $= 2(x-1)\frac{\text{dy}}{\text{d}x}$

Answer: A

Solution:

All straight lines through $(1, -1)$ can be written as

$$y + 1 = m(x - 1),$$

where m is the slope (parameter).

Differentiate w.r.t. x:

$$\frac{dy}{dx} = m.$$

Eliminate m by substituting $m = \frac{dy}{dx}$ back into the family:

$$y + 1 = \left(\frac{dy}{dx}\right)(x - 1) \implies \boxed{y = (x - 1)\frac{dy}{dx} - 1}.$$

Question54

The principal increases continuously in a newly opened bank at the rate of 10% per year. An amount of Rs. 2000 is deposited with this bank. How much will it become after 5 years? $(e^{0.5} = 1.648)$ MHT CET 2025 (19 Apr Shift 2)

Options:

A. 3926

B. 3296

C. 3692

D. 3269

Answer: B

Solution:

For continuous compounding, the amount after time t is

$$A = P\,e^{rt}.$$

Here $P = 2000$, $r = 10\% = 0.10$, $t = 5$ years, so $rt = 0.5$.

$$A = 2000\,e^{0.5}.$$

Given $e^{0.5} = 1.648$,

$$A = 2000 \times 1.648 = 3296.$$

Answer: 3296 (option B).

Question55

The solution of $\frac{\mathrm{dy}}{\mathrm{d}x} = (x + \mathrm{y})^2$ is MHT CET 2025 (19 Apr Shift 2)

Options:

A. $\tan^{-1}(x + y) = x + c,$ where c is the constant of integration

B. $x+\mathrm{y}=\tan x+\mathrm{c}$, where c is the constant of integration

C. $x+\mathrm{y}=\cot^{-1} x+\mathrm{c}$, where c is the constant of integration

D. $x+\mathrm{y}=\sin^{-1}(x+\mathrm{y})+\mathrm{c}$, where c is the constant of integration

Answer: A

Solution:

Let $\frac{dy}{dx}=(x+y)^2$.

Set $z=x+y$. Then

$$\frac{dz}{dx}=1+\frac{dy}{dx}=1+z^2.$$

Separate and integrate:

$$\frac{dz}{1+z^2}=dx \quad \Rightarrow \quad \tan^{-1} z=x+C.$$

Substitute back $z=x+y$:

$$\boxed{\tan^{-1}(x+y)=x+C.}$$

Question56

The order and degree of differential equation of all tangent lines to the parabola $x^2=4\mathrm{y}$ is respectively. MHT CET 2025 (19 Apr Shift 1)

Options:

A. 1,2

B. 2,2

C. 3,1

D. 4,1

Answer: A

Solution:

For the parabola $x^2 = 4y$ (i.e., $y = \frac{x^2}{4}$):

At a point of contact (x, y) on the curve, the slope of the tangent is

$$y' = \frac{dy}{dx} = \frac{x}{2}.$$

Eliminate x between the curve and this slope:

$$x = 2y' \quad \Rightarrow \quad y = \frac{x^2}{4} = \frac{(2y')^2}{4} = (y')^2.$$

Thus the differential equation satisfied by all tangents is

$$y - (y')^2 = 0.$$

The highest derivative is first order ⇒ **order** = **1**; it occurs as a square ⇒ **degree** = **2**.

So, the pair is $\boxed{(1,\ 2)}$.

Question57

If $\mathrm{y} = \mathrm{y}(x)$ satisfies $\left(\frac{2+\sin x}{1+y}\right) \frac{\mathrm{dy}}{\mathrm{d}x} = -\cos x$. such that $y(0) = 2$, then $y\left(\frac{\pi}{2}\right)$ is equal to MHT CET 2025 (19 Apr Shift 1)

Options:

A. 4

B. 3

C. 2

D. 1

Answer: D

Solution:

We're given

$$\frac{2+\sin x}{1+y}\frac{dy}{dx}=-\cos x, \qquad y(0)=2.$$

This is separable:

$$\frac{1}{1+y}\,dy=-\frac{\cos x}{2+\sin x}\,dx.$$

Integrate both sides. Left:

$$\int \frac{1}{1+y}\,dy=\ln(1+y).$$

Right: let $u=2+\sin x \Rightarrow du=\cos x\,dx$:

$$\int -\frac{\cos x}{2+\sin x}\,dx=-\int \frac{1}{u}\,du=-\ln(2+\sin x).$$

So

$$\ln(1+y)=-\ln(2+\sin x)+C \;\Rightarrow\; \ln\left((1+y)(2+\sin x)\right)=C.$$

Hence

$$(1+y)(2+\sin x)=K.$$

Use $y(0)=2$: $(1+2)(2+\sin 0)=3\cdot 2=6 \Rightarrow K=6.$

Thus

$$y=\frac{6}{2+\sin x}-1.$$

Evaluate at $x=\frac{\pi}{2}$ (so $\sin x=1$):

$$y\left(\frac{\pi}{2}\right)=\frac{6}{3}-1=1.$$

$$\boxed{1} \quad \text{(option D)}$$

Question58

In a bank, the principal increases continuously at a rate of $x\%$ per year. Then the rate, x, if ₹100 double itself in 10 years, is$(\log 2=0.6931)$ MHT CET 2025 (19 Apr Shift 1)

Options:

A. 6.93%

B. $9\cdot 63\%$

C. 6.09%

D. 3.69%

Answer: A

Solution:

Use continuous compounding: $A = Pe^{rt}$.

Doubling in 10 years means

$$2 = \frac{A}{P} = e^{r \cdot 10} \quad \Rightarrow \quad r = \frac{\ln 2}{10}.$$

Given $\log_e 2 = 0.6931$,

$$r = \frac{0.6931}{10} = 0.06931.$$

Convert to percent: $x = 100r = 6.931\% \approx \boxed{6.93\%}$ (option A).

Question59

The solution of the differential equation $x \frac{d^2y}{dx^2} = 1$ at $x = y = 1$ with $\frac{dy}{dx} = 0$ at $x = 1$, is MHT CET 2025 (19 Apr Shift 1)

Options:

A. $y = x \log x + x + 2$

B. $y = x \log x - x + 2$

C. $y = x \log x + 2$

D. $x \log x - x = y$

Answer: B

Solution:

Given

$$x \frac{d^2y}{dx^2} = 1, \qquad y(1) = 1, \qquad \left. \frac{dy}{dx} \right|_{x=1} = 0.$$

1. Divide by x:

$$\frac{d^2y}{dx^2} = \frac{1}{x}.$$

2. Integrate once:

$$\frac{dy}{dx} = \int \frac{1}{x}\, dx = \ln x + C_1.$$

Use $\frac{dy}{dx}(1) = 0 \Rightarrow C_1 = 0$. So $\frac{dy}{dx} = \ln x$.

3. Integrate again (by parts):

$$y = \int \ln x \, dx = x \ln x - x + C_2.$$

Use $y(1) = 1$: $1 = 1 \cdot 0 - 1 + C_2 \Rightarrow C_2 = 2$.

Hence,

$$\boxed{y = x \ln x - x + 2}.$$

(Here $\ln$ is the natural logarithm.)

Question60

The slope of tangent at (x, y) to a curve passing through $\left(1, \frac{\pi}{4}\right)$ is $\frac{y}{x} - \cos^2 \frac{y}{x}$, then the equation of curve is MHT CET 2024 (16 May Shift 2)

Options:

A. $y = \tan^{-1}\left(\log\left(\frac{e}{x}\right)\right)$

B. $y = x^2\left(\tan^{-1}\left(\log \frac{e}{x}\right)\right)$

C. $y = x\left(\tan^{-1}\left(\log \frac{e}{x}\right)\right)$

D. $y = \frac{1}{x}\left(\tan^{-1}\left(\log \frac{e}{x}\right)\right)$

Answer: C

Solution:

$\frac{dy}{dx} = \frac{y}{x} - \cos^2 \frac{y}{x} \ldots (i)$
Put $y = vx$...(ii)

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx} \ldots (iii)$
Substituting (ii) and (iii) in (i), we get.

$v + x\frac{dv}{dx} = v - \cos^2 v \quad \Rightarrow x\frac{dv}{dx} = -\cos^2 v$
Integrating on both sides, we get

$$\int \sec^2 v \cdot dv = -\int \frac{dx}{x} + c$$
$$\Rightarrow \tan v = -\log x + c$$
$$\Rightarrow \tan \frac{y}{x} = -\log x + c \ldots (iv)$$

Since the required curve passes through $\left(1, \frac{\pi}{4}\right)$,

$$\tan \frac{\pi}{4} = -\log 1 + c \Rightarrow c = 1$$
$$\therefore \quad \tan \frac{y}{x} = -\log x + 1$$
$$\Rightarrow \tan \frac{y}{x} = -\log x + \log e$$
$$\Rightarrow y = x \tan^{-1}\left[\log\left(\frac{e}{x}\right)\right]$$

Question61

The rate of growth of bacteria in a culture is proportional to the number of bacteria present and the bacteria count is 1000 at $t = 0$. The number of bacteria is increased by

20% in 2 hours. If the population of bacteria is 2000 after $\frac{k}{\log\left(\frac{6}{5}\right)}$ hours, then $\left(\frac{k}{\log 2}\right)^2$ is

MHT CET 2024 (16 May Shift 2)

Options:

A. 16

B. 8

C. 2

D. 4

Answer: D

Solution:

Let ' x ' be the number of bacteria present at time ' t '.

$\therefore \quad \frac{dx}{dt} \propto x$

$\therefore \quad \frac{dx}{dt} = \lambda x$, where λ is the constant of proportionality

Integrating on both sides, we get

$\log x = \lambda t + c$

When $t = 0, x = 1000$

$$\begin{aligned}
\therefore \quad & \log 1000 = 0 + c \\
& \Rightarrow c = \log(1000) \\
\therefore \quad & \log x = \lambda t + \log(1000) \ldots (i) \\
& \text{When } t = 2 \\
& x = 1000 + (20\% \text{ of } 1000) \\
& = 1000 + 200 \\
& = 1200
\end{aligned}$$

$$\begin{aligned}
\therefore \quad & \log 1200 = 2\lambda + \log 1000 \\
& \Rightarrow \lambda = \tfrac{1}{2}\log\left(\tfrac{1200}{1000}\right) = \tfrac{1}{2}\log\left(\tfrac{6}{5}\right) \\
\therefore \quad & \log x = \tfrac{t}{2}\log\left(\tfrac{6}{5}\right) + \log(1000)
\end{aligned}$$

...[From (i)]

When $t = \frac{k}{\log\left(\frac{6}{5}\right)}, x = 2000$

$\therefore \quad \log 2000 = \frac{k}{\log\left(\frac{6}{5}\right)} \times \frac{1}{2}\log\left(\frac{6}{5}\right) + \log(1000)$

$\Rightarrow \log\left(\frac{2000}{1000}\right) = \frac{k}{2} \Rightarrow \log 2 = \frac{k}{2}$

$\Rightarrow$

$\Rightarrow \frac{k}{\log 2} = 2$

$\Rightarrow \left(\frac{k}{\log 2}\right)^2 = 2^2 = 4$

Question62

The general solution of the differential equation $\frac{dy}{dx} = y \tan x - y^2 \sec x$ is MHT CET 2024 (16 May Shift 2)

Options:

A. $\tan x = (c + \sec x)y$, where c is constant of integration.

B. $\sec y = (c + \tan y)x$, where c is constant of integration.

C. $\sec x = (c + \tan x)y$, where c is constant of integration.

D. $\cos y = (c + \tan y)$, where c is constant of integration.

Answer: C

Solution:

$$\frac{dy}{dx} = y \tan x - y^2 \sec x$$
$$\Rightarrow \frac{1}{y^2} \cdot \frac{dy}{dx} + \tan x\left(-\frac{1}{y}\right) = -\sec x \ldots (i)$$
$$\text{Put } v = -\frac{1}{y} \Rightarrow \frac{dv}{dx} = \frac{1}{y^2} \cdot \frac{dy}{dx}$$

$\therefore \quad \frac{dv}{dx} + (\tan x)v = -\sec x \quad \ldots[\text{ From (i) }]$

$\therefore \quad \text{I.F.} = e^{\int \tan x dx} = e^{\log(\sec x)} = \sec x$

$\therefore$ solution of the given equation is

$$v. \sec x = \int -\sec x \cdot \sec x \, dx + c_1$$
$$\Rightarrow v \sec x = -\tan x + c_1$$
$$\Rightarrow -\frac{1}{y} \sec x = -\tan x + c_1$$
$$\Rightarrow \sec x = y(\tan x + c), \text{ where } c = -c_1$$

Question63

Let $y = y(x)$ be the solution of the differential equation $(x \log x)\frac{dy}{dx} + y = 2x \log x (x \geq 1)$ then $y(e)$ is equal to MHT CET 2024 (16 May Shift 1)

Options:

A. 2

B. 2 e

C. e

D. 1

Answer: A

Solution:

$$
\begin{aligned}
&\text{Given, } (x \log x) \frac{\mathrm{d}y}{\mathrm{d}x} + y = 2x \log x \\
&\text{When } x = 1, y = 0 \\
&(x \log x) \frac{\mathrm{d}y}{\mathrm{d}x} + y = 2x \log x \\
\therefore\ &\frac{\mathrm{d}y}{\mathrm{d}x} + \frac{y}{x \log x} = 2 \\
\therefore\ &\text{I.F. } = \mathrm{e}^{\int \frac{1}{x \log x} \mathrm{d}x} = \mathrm{e}^{\log(\log x)} = \log x
\end{aligned}
$$

$\therefore$ Solution of the given equation is

$$
\begin{aligned}
&y \cdot \log x = \int 2 \log x \ \mathrm{d}x + \mathrm{c} \\
\therefore\ &y \log x = 2(x \log x - x) + \mathrm{c} \\
&\text{When } x = 1, y = 0 \\
\therefore\ &0 = -2 + \mathrm{c} \Rightarrow \mathrm{c} = 2 \\
\therefore\ &y \log x = 2(x \log x - x) + 2 \\
\therefore\ &y(\mathrm{e}) = 2(\mathrm{e} - \mathrm{e}) + 2 = 2
\end{aligned}
$$

Question64

The order of the differential equation, whose solution is $y = (C_1 + C_2)\,\mathrm{e}^x + C_3 \mathrm{e}^{x + C_4}$, is MHT CET 2024 (15 May Shift 2)

Options:

A. 4

B. 1

C. 3

D. 2

Answer: B

Solution:

$$y = (C_1 + C_2)\, e^x + C_3 e^{x + C_4}$$
$$= Ae^x + Be^x, \text{ where } A = C_1 + C_2 \text{ and } B = C_3 e^{C_4}$$
$$= (A + B)e^x$$
$$= De^x, \text{ where } D = A + B$$

This equation consist of one arbitrary constant.

Question65

The general solution of the differential equation $\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{.e^x + e^{-x}}$ is MHT CET 2024 (15 May Shift 2)

Options:

A. $y = e^{-3x} + c$, where c is a constant of integration.

B. $y = e^x + c$, where c is a constant of integration.

C. $y = e^{3x} + c$, where c is a constant of integration.

D. $y = e^{-x} + c$, where c is a constant of integration.

Answer: C

Solution:

$$\frac{dy}{dx} = \frac{3e^{2x} + 3e^{4x}}{e^x + e^{-x}}$$
$$\Rightarrow \frac{dy}{dx} = \frac{3e^{2x}\left(1 + e^{2x}\right)}{\left(\frac{e^{2x}+1}{e^x}\right)}$$
$$\Rightarrow \frac{dy}{dx} = 3e^{2x} \cdot e^x$$
$$\Rightarrow dy = 3e^{3x}\, dx$$

Integrating on both sides, we get$y = e^{3x} + c$

Question66

The differential equation, having general solution as $Ax^2 + By^2 = 1$, where A and B are arbitrary constants, is MHT CET 2024 (15 May Shift 1)

Options:

A. $xy\frac{d^2y}{dx^2} - x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

B. $xy\frac{d^2y}{dx^2}-x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

C. $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2+y\frac{dy}{dx}=0$

D. $xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

Answer: D

Solution:

$Ax^2+By^2=1$

Differentiating w.r.t. x, we get

$2Ax+2By\frac{dy}{dx}=0\ldots(i)$

Again, differentiating w.r.t. x, we get

$2A+2B\left[\left(\frac{dy}{dx}\right)^2+y\frac{d^2y}{dx^2}\right]=0\ldots(ii)$

Solving (i) and (ii), we get

$xy\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^2-y\frac{dy}{dx}=0$

Question67

A radio active substance has half-life of h days, then its initial decay rate is given by Note that at $t=0, M=m_o$ MHT CET 2024 (15 May Shift 1)

Options:

A. $\frac{m_o}{h}(\log 2)$

B. $(m_0 h)(\log 2)$

C. $-\frac{m_o}{h}(\log 2)$

D. $(-m_0 h)(\log 2)$

Answer: C

Solution:

Let M be the mass of substance at time t . Then,

$$\frac{dM}{dt} = -kM, \text{ where } k > 0$$
$$\Rightarrow \frac{dM}{M} = -kdt$$

Integrating on both sides, we get

$$\log M = -kt + c$$

When $t = 0, M = m_0$

$$\therefore \quad \log m_0 = 0 + c$$
$$\Rightarrow c = \log m_0$$
$$\therefore \quad \log M = -kt + \log m_0$$
$$\Rightarrow \log \frac{M}{m_0} = -kt$$

$$\text{When } t = h, M = \frac{1}{2} m_0$$
$$\therefore \quad \log\left(\frac{\frac{1}{2} m_0}{m_0}\right) = -kh$$
$$\Rightarrow \log \frac{1}{2} = -kh$$
$$\Rightarrow \log 2 = kh$$
$$\Rightarrow k = \frac{\log 2}{h} \ldots (i)$$

Initial decay rate,

$$\frac{dM}{dt} = -km\ m_0$$
$$= \frac{-m_0}{h} \log 2 \ldots [From (i)]$$

Question68

The differential equation of $y = e^x \left(a + bx + x^2\right)$ is MHT CET 2024 (15 May Shift 1)

Options:

A. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 2y = 0$

B. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

C. $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 2e^x + y = 0$

D. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - e^x + 2y = 0$

Answer: C

Solution:

$$y = e^x (a + bx + x^2)$$

$$\therefore \quad \frac{dy}{dx} = e^x (a + bx + x^2) + e^x(b + 2x)$$

$$\Rightarrow \frac{dy}{dx} = y + e^x(b + 2x) \ldots (i)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + e^x(b + 2x) + e^x(2)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{dy}{dx} + \frac{dy}{dx} - y + 2e^x \ldots [From(i)]$$

$$\Rightarrow \frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 2e^x + y = 0$$

Question69

An ice ball melts at the rate which is proportional to the amount of ice at that instant. Half of the quantity of ice melts in 15 minutes. x_0 is the initial quantity of ice. If after 30 minutes the amount of ice left is kx_0, then the value of k is MHT CET 2024 (11 May Shift 2)

Options:

A. $\frac{1}{2}$

B. $\frac{1}{3}$

C. $\frac{1}{4}$

D. $\frac{1}{8}$

Answer: C

Solution:

Given that half of the ice melts in 15 minutes.

$\therefore$ In 30 minutes, $\left(\frac{1}{4}\right)^{\text{th}}$ of the ice will melt.

$\therefore \quad k = \frac{1}{4}$

Question70

Let $y = y(x)$ be the solution of the differential equation $x\frac{dy}{dx} + y = x\log x, (x > 1)$If $2(y(2)) = \log 4 - 1$ then the value of $y(e)$ is MHT CET 2024 (11 May Shift 2)

Options:

A. $\frac{e^2}{4}$

B. $\frac{-e^2}{2}$

C. $\frac{-e}{2}$

D. $\frac{e}{4}$

Answer: D

Solution:

$$x\frac{dy}{dx} + y = x\log x$$
$$\therefore \quad \frac{dy}{dx} + \frac{1}{x}y = \log x$$

Here, $P(x) = \frac{1}{x}, Q(x) = \log x$

$\therefore \quad$ I.F. $= e^{\int P(x)dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$

$\therefore \quad y(\text{ I.F. }) = \int Q(\text{ I.F. })dx + c$

$\therefore \quad xy = \int x\log x\, dx + c$

$\therefore \quad xy = \frac{x^2}{2}\log x - \left(\int \frac{x^2}{2} \times \frac{1}{x}\, dx\right) + c$

$\therefore \quad xy = \frac{x^2}{2}\log x - \frac{1}{2}\int x\, dx + c$

$\therefore \quad xy = \frac{x^2}{2}\log x - \frac{1}{4}x^2 + c \ldots (i)$

Given that $2(y(2)) = \log 4 - 1$

$\therefore \quad y(2) = \log 2 - \frac{1}{2}$

$\therefore \quad$ From equation (i), we get

$2\left(\log 2 - \frac{1}{2}\right) = \frac{(2)^2}{2}\log 2 - \frac{1}{4}(2)^2 + c$

$\therefore \quad 2\log 2 - 1 = 2\log 2 - 1 + c$

$\therefore \quad c = 0 \ldots (ii)$

$\therefore \quad$ From (i) and (ii), we get

$y(e) = \frac{e}{2} - \frac{e}{4} = \frac{e}{4}$

Question71

If $y(\dot{x})$ is the solution of the differential equation $(x+2)\frac{dy}{dx} = x^2 + 4x - 9, x \neq -2$ and $y(0) = 0$, then $y(-4)$ is equal to MHT CET 2024 (11 May Shift 2)

Options:

A. 0

B. 1

C. -1

D. 2

Answer: A

Solution:

$$(x+2)\frac{dy}{dx} = x^2 + 4x - 9$$

$\therefore \quad dy = \frac{(x^2+4x+4)-4-9}{x+2}\,dx$

$\therefore \quad dy = \frac{(x+2)^2-13}{(x+2)}dx$

Integrating both sides, we get

$$\int dy = \int (x+2)dx - 13\int \frac{1}{x+2}\,dx$$

$$y = \frac{(x+2)^2}{2} - 13\log|x+2| + c \ldots (i)$$

Given that $y(0) = 0$

$\therefore$ from equation (i), we get

$\therefore \quad 0$

$\therefore \quad = 2 - 13\log|0+2| + c \ldots (ii)$

$= 13\log(2) - 2$

$\therefore$ from (i) and (ii), we get

$$y(-4) = 2 - 13\log(2) + 13\log(2) - 2 = 0$$

Question72

The bacteria increase at the rate proportional to the number of bacteria present. If the original number N doubles in 8 hours, then the number of bacteria in 24 hours will be MHT CET 2024 (11 May Shift 1)

Options:

A. 8 N

B. 16 N

C. 32 N

D. 64 N

Answer: A

Solution:

Initial number of bacteria $= \mathrm{N}$

$\therefore$ After 8 hours number of bacteria $= 2\mathrm{~N}$

$\therefore$ After 16 hours number of bacteria $= 4\mathrm{~N}$

$\therefore$ After 24 hours number of bacteria $= 8\mathrm{~N}$

Question73

The general solution of $\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$ is MHT CET 2024 (11 May Shift 1)

Options:

A. $\log\tan\left(\frac{y}{2}\right) = C - 2\sin x$

B. $\log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

C. $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = C - 2\sin x$

D. $\log\tan\left(\frac{y}{2} + \frac{\pi}{4}\right) = C - 2\sin\left(\frac{x}{2}\right)$

Answer: B

Solution:

$$\frac{dy}{dx} + \sin\left(\frac{x+y}{2}\right) = \sin\left(\frac{x-y}{2}\right)$$
$$\Rightarrow \frac{dy}{dx} = \sin\left(\frac{x-y}{2}\right) - \sin\left(\frac{x+y}{2}\right)$$
$$\Rightarrow \frac{dy}{dx} = -2\sin\left(\frac{y}{2}\right)\cdot\cos\left(\frac{x}{2}\right)$$

Integrating on both sides, we get

$$\int \operatorname{cosec}\left(\frac{y}{2}\right)dy = -\int 2\cos\left(\frac{x}{2}\right)dx + c_1$$
$$\Rightarrow \frac{\log\tan\left(\frac{y}{4}\right)}{\frac{1}{2}} = -\frac{2\sin\left(\frac{x}{2}\right)}{\frac{1}{2}} + c_1$$
$$\Rightarrow \log\tan\left(\frac{y}{4}\right) = C - 2\sin\left(\frac{x}{2}\right), \text{ where } C = \frac{1}{2}c_1$$

Question74

The particular solution of the differential equation, $xy\frac{dy}{dx} = x^2 + 2y^2$ when $y(1) = 0$ is MHT CET 2024 (11 May Shift 1)

Options:

A. $\frac{x^2+y^2}{x^3} = 1$

B. $x^2+y^2=x$

C. $x^2+y^2=x^4$

D. $x^2+2y^2=x^4$

Answer: C

Solution:

$$xy\frac{dy}{dx}=x^2+2y^2$$
$$\therefore \quad \frac{dy}{dx}=\frac{x^2+2y^2}{xy}\ldots(i)$$

Put $y=vx\ldots(ii)$

Differentiating w.r.t. x, we get

$$\frac{dy}{dx}=v+x\frac{dv}{dx}\ldots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$v+x\frac{dv}{dx}=\frac{x^2+2v^2x^2}{x(vx)}$$
$$\therefore \quad v+x\frac{dv}{dx}=\frac{x^2(1+2v^2)}{x^2v}$$
$$\therefore \quad x\frac{dv}{dx}=\frac{1+2v^2}{v}-v=\frac{1+v^2}{v}$$
$$\therefore \quad \frac{v}{1+v^2}dv=\frac{1}{x}\,dx$$

Integrating on both sides, we get

$$\frac{1}{2}\int\frac{2v}{1+v^2}dv=\int\frac{dx}{x}$$
$$\therefore \quad \frac{1}{2}\log\left|1+v^2\right|=\log|x|+\log|c_1|$$

$$\therefore \quad \log\left|1+v^2\right|=2\log|x|+2\log|c_1|$$
$$=\log\left|x^2\right|+\log|c|\ldots\left[\log c_1^2=\log c\right]$$
$$\therefore \quad \log\left|1+v^2\right|=\log\left|cx^2\right|$$

$$\therefore \quad 1+v^2=cx^2$$
$$\therefore \quad 1+\frac{y^2}{x^2}=cx^2$$
$$\therefore \quad x^2+y^2=cx^4\ldots(iv)$$

Putting $x=1$ and $y=0$, we get

$$1+0=c(1)$$
$$\therefore \quad c=1$$

Substituting $c=1$ in (iv), we get

$$x^2+y^2=x^4$$

Question75

The general solution of the differential equation $e^{y-x}\frac{dy}{dx} = y\left(\frac{\sin x+\cos x}{1+y\log y}\right)$. is MHT CET 2024 (10 May Shift 2)

Options:

A. $e^{y}\log y = e^{x}\sin x + c$, where c is a constant of integration.

B. $e^{y} = e^{x}\sin x + c$, where c is a constant of integration.

C. $\log y = e^{x}\sin x + c$, where c is a constant of integration.

D. $y\log y = e^{x}\sin x + c$, where c is a constant of integration.

Answer: A

Solution:

$$e^{y-x}\frac{dy}{dx} = y\left(\frac{\sin x+\cos x}{1+y\log y}\right)$$

$$\therefore \quad \frac{e^{y}}{e^{x}}\frac{dy}{dx} = \frac{y}{(1+y\log y)}(\sin x+\cos x)$$

$$\therefore \quad e^{y}\frac{(1+y\log y)}{y}\,dy = e^{x}(\sin x+\cos x)dx$$

$$\therefore \quad e^{y}\left(\log y+\frac{1}{y}\right)dy = e^{x}(\sin x+\cos x)dx$$

Integrating both sides, we get

$$e^{y}\log y = e^{x}\sin x + c$$

$$\ldots\left[\because \int e^{x}\left[f(x)+f'(x)\right]dx = e^{x}f(x)+c\right]$$

Question76

A spherical rain drop evaporates at a rate proportional to its surface area. If initially its radius is 3 mm and after 1 second it is reduced to 2 mm , then at any time t its radius is (where $0 \leq t < 3$) MHT CET 2024 (10 May Shift 2)

Options:

A. $3+t$

B. $3-t$

C. $4-t$

D. $1+t$

Answer: B

Solution:

$$\frac{dv}{dt} \propto -s$$

$$\therefore \quad \frac{dv}{dt} = -ks, \ldots (i)$$

where $k > 0$

$$v = \frac{4}{3}\pi r^3 \text{ and } s = 4\pi r^2$$

$$\Rightarrow \frac{dv}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$$

Equation (i) becomes

$$4\pi r^2 \frac{dr}{dt} = -k\left(4\pi r^2\right)$$

$$\Rightarrow \frac{dr}{dt} = -k$$

Integrating on both sides, we get

$$r = -kt + c \ldots (ii)$$

When $t = 0, r = 3$

$$\therefore \quad 3 = -k(0) + c \Rightarrow c = 3$$

$$\therefore \quad r = -kt + 3 \ldots [From(ii)]$$

When $t = 1, r = 2$

$$\therefore \quad 2 = -k(1) + 3 \Rightarrow k = 1$$

$$\therefore \quad r = -t + 3$$

$$\Rightarrow r = 3 - t$$

Question77

The order of the differential equation, whose general solution is given by $y = (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c^5}$ where c_1, c_2, c_3, c_4 and c_5 are arbitrary constant, is MHT CET 2024 (10 May Shift 2)

Options:

A. 5

B. 3

C. 4

D. 2

Answer: B

Solution:

$$\begin{aligned} y = & (c_1 + c_2)\cos(x + c_3) - c_4 e^{x + c^5} \\ = & (c_1 + c_2)\cos(x + c_3) - c_4 e^x e^{c_5} \\ = & A\cos(x + c_3) - Be^x \\ & \ldots [\mathrm{A} = c_1 + c_2, B = c_4 e^{c_5}] \end{aligned}$$

There are 3 arbitrary constants. $\therefore$ Order of the given differential equation is 3.

Question78

If $\cos x \frac{dy}{dx} - y \sin x = 6x, 0 < x < \frac{\pi}{2}$, then general solution of the differential equation is MHT CET 2024 (10 May Shift 1)

Options:

A. $y = \cos x + 3x^2 + c$, where c is a constant of integration.

B. $y + \cos x = 3x^2 + c$, where c is a constant of integration.

C. $y = 3x^2 \cos x + \cos x$, where c is a constant of integration.

D. $y \cdot \cos x = 3x^2 + c$, where c is a constant of integration.

Answer: D

Solution:

$$\cos x \frac{dy}{dx} - y \sin x = 6x$$
$$\Rightarrow \frac{dy}{dx} - (\tan x) y = 6x \sec x$$

This equation is of the form $\frac{dy}{dx} + py = Q$

$\therefore \quad \text{If} = e^{\int p dx} = e^{-\int \tan x dx} = e^{\log x \cos x} = \cos x$

$\therefore$ Solution of given equation is

$$y \cdot \cos x = \int 6x \times \sec x \times \cos x \, dx + c$$
$$\Rightarrow y \cos x = \int 6x \, dx + c$$
$$\Rightarrow y \cos x = 3x^2 + c$$

Question79

The general solution of $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is MHT CET 2024 (10 May Shift 1)

Options:

A. $y = x + \log(x + y) + c$, where c is a constant of integration.

B. $y = x - \log(x + y) + c$, where c is a constant of integration.

C. $y = x - \log(2x + y) + c$, where c is a constant of integration.

D. $y = x^2 + \log(x + y) + c$, where c is a constant of integration.

Answer: A

Solution:

$\frac{dy}{dx} = \frac{x+y+1}{x+y-1} \ldots (i)$

Put $x + y = v \ldots$(ii)

$\Rightarrow \frac{dy}{dx} = \frac{dv}{dx} - 1 \ldots (iii)$

Substituting (ii) and (iii) in (i), we get

$$\frac{dv}{dx} - 1 = \frac{v+1}{v-1}$$
$$\Rightarrow \frac{dv}{dx} = \frac{2v}{v-1} \Rightarrow \frac{v-1}{2v} dv = dx$$

Integrating on both sides, we get

$$\frac{v}{2} - \frac{1}{2} \log v = x + c_1$$
$$\Rightarrow v - \log v = 2x + 2c_1$$
$$\Rightarrow x + y - \log(x+y) = 2x + 2c_1$$
$$\Rightarrow y = x + \log(x+y) + c, \text{ where } c = 2c_1$$

Question80

A radio-active substance has a half-life of h days, then its initial decay rate is given by (where radio-active substance has initial mass m_0). MHT CET 2024 (10 May Shift 1)

Options:

A. $\frac{m_0}{h}(\log 2)$

B. $(m_0 h)(\log 2)$

C. $-\frac{m_0}{h}(\log 2)$

D. $-(m_0 h)(\log 2)$

Answer: C

Solution:

Let m be the mass of substance at time t. Then,

$$\frac{\mathrm{dm}}{\mathrm{dt}}=-\mathrm{km}, \text{ where } \mathrm{k}>0$$
$$\Rightarrow \frac{\mathrm{dm}}{\mathrm{m}}=-\mathrm{kdt}$$

Integrating on both sides, we get

$$\log m=-kt+c$$

When $\mathrm{t}=0,\ \mathrm{m}=\mathrm{m}_0$

$$\therefore \quad \log \mathrm{m}_0=0+c$$
$$\Rightarrow \mathrm{c}=\log \mathrm{m}_0$$
$$\therefore \quad \log\ \mathrm{m}=-\mathrm{kt}+\log \mathrm{m}_0$$
$$\Rightarrow \log \frac{\mathrm{m}}{\mathrm{m}_0}=-\mathrm{kt}$$

$$\text{When } \mathrm{t}=\mathrm{h},\ \mathrm{m}=\frac{1}{2}\mathrm{m}_0$$
$$\therefore \quad \log\left(\frac{\frac{1}{2}\mathrm{m}_0}{\mathrm{m}_0}\right)=-\mathrm{kh}$$
$$\Rightarrow \log \frac{1}{2}=-\mathrm{kh}$$
$$\Rightarrow \log 2=\mathrm{kh}$$
$$\Rightarrow \mathrm{k}=\frac{\log 2}{\mathrm{h}} \ldots(i)$$

Initial decay rate

$$\frac{\mathrm{dm}}{\mathrm{dt}}=-\mathrm{km}\ \mathrm{m}_0$$
$$=\frac{-\mathrm{m}_0}{\mathrm{h}}\log 2 \ldots[From(i)]$$

Question81

The general solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y+\sqrt{x^2-y^2}}{x}$ is MHT CET 2024 (09 May Shift 2)

Options:

A. $\sin^{-1} y=\log x+\mathrm{c}$, where c is a constant of integration.

B. $\frac{y}{x}=\sin^{-1} x+\mathrm{c}$, where c is a constant of integration.'

C. $\frac{y}{x}=\sqrt{x^2-y^2}+\mathrm{c}$, where c is a constant of integration.

D. $\sin^{-1}\left(\frac{y}{x}\right)=\log x+\mathrm{c}$, where c is a constant of integration.

Answer: D

Solution:

$$\frac{dy}{dx}=\frac{y+\sqrt{x^2-y^2}}{x}\ldots(i)$$

Put $y=vx\ldots(ii)$

$$\therefore \frac{dy}{dx}=v+x\frac{dv}{dx}\ldots(iii)$$

Substituting (ii) and (iii) in (i), we get

$$v+x\frac{dv}{dx}=\frac{vx+\sqrt{x^2-v^2x^2}}{x}$$

$$\therefore v+x\frac{dv}{dx}=v+\sqrt{1-v^2}$$

$$\therefore x\frac{dv}{dx}=\sqrt{1-v^2}$$

$$\therefore \int\frac{dv}{\sqrt{1-v^2}}=\int\frac{dx}{x}$$

$$\therefore \sin^{-1}(v)=\log|x|+c$$

$$\therefore \sin^{-1}\left(\frac{y}{x}\right)=\log|x|+c$$

Question82

In a certain culture of bacteria, the rate of increase is proportional to the number present. If there are 10^4 at the end of 3 hours and $4\cdot10^4$ at the end of 5 hours, then there were ______ in the beginning. MHT CET 2024 (09 May Shift 2)

Options:

A. 10^4

B. $\frac{10^4}{4^1}$

C. 410^4

D. $\frac{10^4}{8}$

Answer: D

Solution:

Let x be the number of bacteria present at time t.

$\therefore \quad \frac{dx}{dt} \propto x$
$\therefore \quad \frac{dx}{dt} = kx$
$\therefore \quad \frac{dx}{x} = kdt$

Integrating on both sides, we get

$\log x = kt + c$...(i)

When $t = 3, x = 10^4 = 10,000$

Equation (i) becomes

$\log(10,000) = 3k + c$

...(ii)

When $t = 5, x = 4.10^4 = 40,000$

Equation (i) becomes

$\log(40,000) = 5k + c$...(iii)

Subtracting (ii) from (iii), we get

$k = \log 2$
From equation (ii),

$\log(10;000) = 3\log 2 + c$

$\therefore \quad c = \log\left(\frac{10^4}{8}\right)$
Now, Initially $t = 0$

From (i),

$$\log x = k \times 0 + \log\left(\frac{10^4}{8}\right)$$
$$\Rightarrow \log x = \log\left(\frac{10^4}{8}\right)$$
$$\therefore \quad x = \frac{10^4}{8}$$

Question83

Integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$ is MHT CET 2024 (09 May Shift 2)

Options:

A. $\frac{x}{e^x}$

B. xe^x

C. e^x

D. $\frac{e^x}{x}$

Answer: D

Solution:

$$\begin{aligned} & \frac{dy}{dx}+y=\frac{1+y}{x} \\ & \Rightarrow \frac{dy}{dx}+y=\frac{1}{x}+\frac{y}{x} \\ & \Rightarrow \frac{dy}{dx}+\left(1-\frac{1}{x}\right)y=\frac{1}{x} \\ \therefore \quad & \text{I.F. } =e^{\int\left(1-\frac{1}{x}\right)dx}=e^{x-\log x}=\frac{e^x}{x} \end{aligned}$$

Question84

If $y=y(x)$ is the solution of the differential equation $x\frac{dy}{dx}+2y=x^2$ satisfying $y(1)=1$, then the value of $y\left(\frac{1}{2}\right)$ is MHT CET 2024 (09 May Shift 1)

Options:

A. $\frac{7}{64}$

B. $\frac{1}{4}$

C. $\frac{13}{6}$

D. $\frac{49}{16}$

Answer: D

Solution:

$$x\frac{dy}{dx}+2y=x^2$$
$$\frac{dy}{dx}+\left(\frac{2}{x}\right)y=x$$
$$\text{I.F.}=e^{\int\frac{2}{x}dx}=e^{2\log x}=x^2$$

$\therefore$ Solution of differential equation is

$$y\cdot x^2=\int x\cdot x^2\,dx+c$$
$$yx^2=\frac{x^4}{4}+c\ldots(i)$$
$\therefore$
$$y=\frac{x^2}{4}+\frac{c}{x^2}\ldots(ii)$$

Given, $y(1)=1$
$$\Rightarrow 1=\frac{1}{4}+c$$
$$\Rightarrow c=\frac{3}{4}$$

$\therefore$ equation (i) becomes,

$$y=\frac{x^2}{4}+\frac{3}{4x^2}$$
$$\therefore\ y\left(\frac{1}{2}\right)=\frac{\left(\frac{1}{2}\right)^2}{4}+\frac{3}{4\times\left(\frac{1}{2}\right)^2}=\frac{1}{16}+3=\frac{49}{16}$$

Question85

The curve satisfying the differential equation $y\,dx-\left(x+3y^2\right)dy=0$ and passing through the point $(1,1)$ also passes through the point MHT CET 2024 (09 May Shift 1)

Options:

A. $\left(\frac{1}{4},\frac{1}{2}\right)$

B. $\left(\frac{1}{4},-\frac{1}{2}\right)$

C. $\left(\frac{1}{3},-\frac{1}{3}\right)$

D. $\left(-\frac{1}{3},\frac{1}{3}\right)$

Answer: D

Solution:

$$y\,\mathrm{d}x-\left(x+3y^2\right)\mathrm{d}y=0$$
$$\Rightarrow y\,\mathrm{d}x=\left(x+3y^2\right)\mathrm{d}y$$
$$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\frac{x+3y^2}{y}$$
$$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}=\frac{x}{y}+3y$$
$$\Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y}-\left(\frac{1}{y}\right)x=3y$$

Which is a linear equation

$\therefore \quad \mathrm{IF}=\mathrm{e}^{\int \frac{-1}{y}\mathrm{d}y}=\mathrm{e}^{-\log y}=\frac{1}{y}$

$\therefore$ The required solution is

$x\frac{1}{y}=\int 3y\times\frac{1}{y}\,\mathrm{d}y+\mathrm{c}$

$$\therefore \quad \frac{x}{y}=3y+c$$
$$\Rightarrow x=3y^2+cy$$

Curve passes through $(1,1)$

$$\Rightarrow 1=3+c$$
$$\Rightarrow c=-2$$

Equation (i) becomes,

$x=3y^2-2y$

Option (D) i.e., $\left(\frac{-1}{3},\frac{1}{3}\right)$

Satisfies above equation.

Question86

The general solution of the differential equation $\frac{1}{x}\frac{\mathrm{d}y}{\mathrm{d}x}=\tan^{-1}x$ is MHT CET 2024 (04 May Shift 2)

Options:

A. $y+\frac{x^2\tan^{-1}x}{2}+\mathrm{c}=0$, where c is a constant of integration.

B. $y+x\tan^{-1}x+\mathrm{c}=0$, where c is a constant integration.

C. $y-x-\tan^{-1}x+\mathrm{c}=0$, where is a constant of integration.

D. $y=\frac{x^2\tan^{-1}x}{2}-\frac{1}{2}\left(x-\tan^{-1}x\right)+\mathrm{c}$, where c is constant of integration.

Answer: D

Solution:

$$\frac{1}{x}\frac{dy}{dx} = \tan^{-1}x$$
$$dy = x \cdot \tan^{-1}x\, dx$$
Integrating on both sides we get,

$$y = \int x \cdot \tan^{-1}x\, dx$$
$$y = \tan^{-1}x \int x\, dx - \int\left(\frac{d}{dx}\tan^{-1}x \cdot \int x\, dx\right) dx$$
$$y = \tan^{-1}x \cdot \frac{x^2}{2} - \int\left(\frac{1}{1+x^2} \times \frac{x^2}{2}\right) dx$$
$$y = \tan^{-1}x \cdot \frac{x^2}{2} - \frac{1}{2}\int\left(\frac{x^2}{1+x^2}\right) dx$$
$$= \frac{x^2 \cdot \tan^{-1}x}{2} - \frac{1}{2}\left(\int \frac{x^2+1}{x^2+1} - \int \frac{1}{1+x^2}\, dx\right)$$
$$\Rightarrow y = \frac{x^2 \tan^{-1}x}{2} - \frac{1}{2}(x - \tan^{-1}x) + c$$

Question87

The differential equation obtained by eliminating arbitrary constant from the equation $y^2 = (x+c)^3$ is MHT CET 2024 (04 May Shift 2)

Options:

A. $\left(\frac{dy}{dx}\right)^3 = 27y$

B. $\left(\frac{dy}{dx}\right)^3 = -27y$

C. $8\left(\frac{dy}{dx}\right)^3 = 27y$

D. $8\left(\frac{dy}{dx}\right)^3 + 27y = 0$

Answer: C

Solution:

$y^2=(x+c)^3$

Differentiating w.r.to x, we get

$$2y\frac{\mathrm{d}y}{\mathrm{d}x}=3(x+\mathrm{c})^2$$

$$\Rightarrow(x+\mathrm{c})^2=\frac{2y}{3}\frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\Rightarrow(x+\mathrm{c})^6=\left(\frac{2y}{3}\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

$$\Rightarrow\left((x+\mathrm{c})^3\right)^2=\frac{8y^3}{27}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

$$\Rightarrow\left(y^2\right)^2=\frac{8y^3}{27}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

$$\Rightarrow y^4=\frac{8y^3}{27}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

$$\Rightarrow 27y=8\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^3$$

Question88

The decay rate of radium is proportional to the amount present at any time t. If initially 60 gms was present and half life period of radium is 1600 years, then the amount of radium present after 3200 years is MHT CET 2024 (04 May Shift 2)

Options:

A. 20 grams

B. 15 grams

C. 12 grams

D. 10 grams

Answer: B

Solution:

Let m be the mass of substance at time t .

Then

$$\frac{\mathrm{dm}}{\mathrm{dt}}=-\mathrm{km}, \text{ where } \mathrm{k}>0$$
$$\Rightarrow \frac{\mathrm{dm}}{\mathrm{m}}=-\mathrm{kdt}$$

Integrating on both sides, we get

$$\log m=-kt+c$$

when $\mathrm{t}=0, \mathrm{~m}=60 \mathrm{gms}$

$$\begin{aligned}
\therefore \quad & \log 60=-k(0)+c \\
& \Rightarrow \mathrm{c}=\log 60 \\
& \Rightarrow \mathrm{c}=\log 60 \\
\therefore \quad & \log \mathrm{~m}=-\mathrm{kt}+\log 60 \\
\therefore \quad & \text{when } \mathrm{t}=1600, \mathrm{~m}=\tfrac{60}{2}=30 \mathrm{gms} \\
\therefore \quad & \log 30=-1600 \mathrm{k}+\log 60 \\
& \Rightarrow 1600 \mathrm{k}=\log 2 \\
& \Rightarrow \mathrm{k}=\tfrac{1}{1600} \log 2
\end{aligned}$$

Equation (i) becomes

$$\log m=\tfrac{-1}{1600}(\log 2) t+\log 60$$

When $t=3200$ years

$$\begin{aligned}
& \log \mathrm{m}=\frac{-1}{1600} \times \log 2 \times 3200+\log 60 \\
& \log \mathrm{~m}=-\log 4+\log 60 \\
& \Rightarrow \log \mathrm{~m}=\log \frac{60}{4}
\end{aligned}$$

$$\Rightarrow \mathrm{m}=15 \text{ grams}$$

Question89

The particular solution of differential equation $\left(1+y^2\right)(1+\log x) \mathrm{d} x+x \mathrm{~d} y=0$ at $x=1, y=1$ is MHT CET 2024 (04 May Shift 2)

Options:

A. $\log x-\frac{1}{2}(\log x)^2-\tan^{-1} y=-\frac{\pi}{4}$

B. $\log x+\frac{1}{2}(\log x)^2+\tan^{-1} y=\frac{\pi}{4}$

C. $\log x-\frac{1}{2}(\log x)^2+\tan^{-1} y=\frac{\pi}{4}$

D. $\log x+\frac{1}{2}(\log x)^2-\tan^{-1} y=\frac{\pi}{4}$

Answer: B

Solution:

$$
\begin{aligned}
&\left(1+y^2\right)(1+\log x)\mathrm{d}x+x\,\mathrm{d}y=0\\
&\Rightarrow\left(1+y^2\right)(1+\log x)\mathrm{d}x=-x\,\mathrm{d}y\\
&\Rightarrow\left(\frac{1+\log x}{x}\right)\mathrm{d}x=\left(\frac{-1}{1+y^2}\right)\mathrm{d}y
\end{aligned}
$$

Integrating on both sides, we get

$$\Rightarrow\int\frac{(1+\log x)}{x}\,\mathrm{d}x=-1\int\frac{1}{1+y^2}\,\mathrm{d}y$$

$$\Rightarrow\int \mathrm{t}\mathrm{dt}=-\tan^{-1}y+\mathrm{c}\quad\cdots\left[\begin{array}{l}\text{Let }1+\log x=\mathrm{t}\\ \frac{1}{x}\,\mathrm{d}x=\mathrm{dt}\end{array}\right]$$

$$
\begin{aligned}
&\Rightarrow\frac{\mathrm{t}^2}{2}=-\tan^{-1}y+\mathrm{c}\ldots(i)\\
&\Rightarrow\frac{(1+\log x)^2}{2}=-\tan^{-1}y+\mathrm{c}
\end{aligned}
$$

$$
\begin{aligned}
&\text{At }x=1,y=1\\
&\Rightarrow\frac{(1+\log 1)^2}{2}=-\tan^{-1}(1)+c\\
&\Rightarrow c=\frac{1}{2}+\frac{\pi}{4}
\end{aligned}
$$

Substituting above value in (i), we get

$$
\begin{aligned}
&\frac{(1+\log x)^2}{2}=-\tan^{-1}y+\frac{1}{2}+\frac{\pi}{4}\\
&\frac{1}{2}+\log x+\frac{(\log x)^2}{2}=-\tan^{-1}y+\frac{1}{2}+\frac{\pi}{4}\\
&\Rightarrow\log x+\frac{(\log x)^2}{2}+\tan^{-1}y=\frac{\pi}{4}
\end{aligned}
$$

Question90

Let $y=y(x)$ be the solution of the differential equation $\sin x\frac{\mathrm{d}y}{\mathrm{d}x}+y\cos x=4x, x\in(0,\pi)$.If $y\left(\frac{\pi}{2}\right)=0$, then $y\left(\frac{\pi}{6}\right)$ is equal to MHT CET 2024 (04 May Shift 1)

Options:

A. $-\frac{4}{9}\pi^2$

B. $\frac{4}{9\sqrt{3}}\pi^2$

C. $\frac{-8}{9\sqrt{3}}\pi^2$

D. $-\frac{8}{9}\pi^2$

Answer: D

Solution:

$$\sin x \frac{dy}{dx} + y\cos x = 4x$$

$$\therefore \quad \frac{dy}{dx} + y\cot x = \frac{4x}{\sin x}$$

Here, $P(x) = \cot x, Q(x) = \frac{4x}{\sin x}$

Integrating factor (I.F.) $= e^{\int P(x)dx}$

$= e^{\int \cot x dx}, = e^{\log|\sin x|} = \sin x$

$$\therefore \quad y(\text{ I.F. }) = \int Q(\text{ I.F. })dx + c$$

$$\therefore \quad y(\sin x) = \int \tfrac{4x}{\sin x} \times \sin x \, dx = C$$

$$\therefore \quad y\sin x = 4\int x \, dx + C \qquad \text{...(i)}$$

$$\therefore \quad y = \tfrac{2x^2 + C}{\sin x}$$

$$y\left(\tfrac{\pi}{2}\right) = 0$$

$$\Rightarrow \frac{2\left(\frac{\pi}{2}\right)^2 + C}{\sin\left(\frac{\pi}{2}\right)} = 0$$

$$\Rightarrow \frac{\pi^2}{2} + C = 0$$

$$\Rightarrow C = \frac{-\pi^2}{2}$$

$$\Rightarrow y = \tfrac{2x^2 - \frac{\pi^2}{2}}{\sin x}$$

is the solution of the given differential equation.

$$\therefore y\left(\frac{\pi}{6}\right) = \frac{2\left(\frac{\pi}{6}\right)^2 - \frac{\pi^2}{2}}{\sin\left(\frac{\pi}{6}\right)}$$

$$= \frac{\frac{2\pi^2}{36} - \frac{\pi^2}{2}}{\frac{1}{2}} = \frac{-8}{9}\pi^2$$

Question91

Given that the slope of the tangent to a curve $y = y(x)$ at any point (x, y) is $\frac{2y}{x^2}$. If the curve passes through the centre of the circle $x^2 + y^2 - 2x - 2y = 0$, then its equation is MHT CET 2024 (04 May Shift 1)

Options:

A. $x\log|y| = x - 1$

B. $x\log|y| = -2(x - 1)$

C. $x\log|y| = 2(x - 1)$

D. $x^2\log|y| = -2(x - 1)$

Answer: C

Solution:

Equation of the given circle is

$x^2+y^2-2x-2y=0$

$\therefore \quad x^2-2x+1+y^2-2y+1=2$

$\therefore \quad (x-1)^2+(y-1)^2=2$

$\therefore \quad$ Centre of the circle is $(1,1)$

Now, slope of the given tangent is $\frac{2y}{x^2}$

i.e., $\frac{dy}{dx}=\frac{2y}{x^2}$.

$\therefore$ Integrating on both sides, we get

$\int \frac{1}{y}\, dy = 2\int x^{-2}\, dx$

$\therefore \quad \log|y| = \frac{-2}{x}+c$...(i)

At $(1,1)$, we get

$\log 1 = -2+c$

$\therefore \quad c=2$

$\therefore$ Required equation is

$\log|y| = \frac{-2}{x}+2$

i.e., $x\log|y| = 2(x-1)$

Question92

The general solution of the differential equation $x\cos y\, dy = (xe^x\log x + e^x)\, dx$ is given by MHT CET 2024 (03 May Shift 2)

Options:

A. $\sin y = e^x + \operatorname{cog} x$, where c is a constant of integration.

B. $\sin y = e^x\log x + c$, where c is a constant of integration.

C. $e^x\sin y = \log x + c$, where c is a constant of integration,

D. $\sin y = ce^x + \log x$, where c is a constant of integration.

Answer: B

Solution:

$$x \cos y \, \mathrm{d}y = (x\mathrm{e}^x \log x + \mathrm{e}^x) \, \mathrm{d}x$$
$$\Rightarrow \cos y \, \mathrm{d}y = \mathrm{e}^x \left(\log x + \frac{1}{x}\right) \mathrm{d}x$$

Integrating on both sides, we get

$$\sin y = \mathrm{e}^x \log x + \mathrm{c}$$

Question93

If order and degree of the differential equation $\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5 + 4\frac{\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5}{\left(\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}\right)} + \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \sin x$, are m and n respectively, then the value of $(m^2 + n^2)$ is equal to MHT CET 2024 (03 May Shift 2)

Options:

A. 29

B. 13

C. 5

D. 8

Answer: B

Solution:

Given differential equation is

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5 + 4\frac{\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5}{\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}} + \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} = \sin x$$

$$\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5 \cdot \frac{\mathrm{d}^3 y}{\mathrm{d}x^3} + 4\left(\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}\right)^5 + \left(\frac{\mathrm{d}^3 y}{\mathrm{d}x^3}\right)^2 = \sin x \frac{\mathrm{d}^3 y}{\mathrm{d}x^3}$$

Here, order $= 3$ and degree $= 2$

$$\therefore \quad m = 3, n = 2$$
$$\therefore \quad m^2 + n^2 = 3^2 + 2^2 = 13$$

Question94

The differential equation $\left[\frac{1+\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}}\right]^2 = kx$ **is of MHT CET 2024 (03 May Shift 1)**

Options:

A. order $=2$, degree $=3$

B. order $=3$, degree $=2$

C. order $=2$, degree $=2$

D. order $=3$, degree $=3$

Answer: A

Solution:

$$\left[\frac{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}\right]^{\frac{3}{2}} = \mathrm{k}x$$

$$\therefore \left[\frac{1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}{\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)}\right]^{3} = (\mathrm{kx})^2$$

$$\therefore \quad \left[1+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2\right]^3 = (\mathrm{k}x)^2\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^3$$

$$\therefore \quad \text{Order } = 2, \text{ Degree } = 3$$

Question95

If the half life of substance is 5 years, then the total amount of the substance left after 15 years, when initial amount is 64 gms is MHT CET 2024 (03 May Shift 1)

Options:

A. 6 gm

B. 8 gm

C. 10 gm

D. 12 gm

Answer: B

Solution:

Initial amount $= 64\text{gms}$

Half life period $= 5$: years

$\therefore$ Amount of substance left after 5 years $= \frac{64}{2} = 32\text{gms}$

Amount of substance left after 10 years $= \frac{32}{2} = 16\text{gms}$

Amount of substance left after 15 years $= \frac{16}{2} = 8\text{gms}$

Question96

A body cools according to Newton's law of cooling from 100°C to 60°C in 15 minutes. If the temperature of the surrounding is 20°C, then the temperature of the body after cooling down for one hour is MHT CET 2024 (02 May Shift 2)

Options:

A. 30°C

B. 25°C

C. 35°C

D. 40°C

Answer: B

Solution:

Let θ be the temperature of the body at any time t.

$$\therefore \quad \frac{d\theta}{dt} \propto (\theta - 20)$$
$$\Rightarrow \frac{d\theta}{dt} = -k(\theta - 20), k > 0$$

Integrating on both sides, we get

$$\log|\theta - 20| = -kt + c$$
When $t = 0, \theta = 100^\circ$
$$\therefore \quad \log 80 = -k(0) + c$$
$$\Rightarrow c = \log 80$$

$$\therefore \quad \log|\theta - 20| = -kt + \log 80$$
When $t = 15, \theta = 60^\circ$
$$\therefore \quad \log 40 = -15k + \log 80$$
$$\Rightarrow \quad k = \tfrac{-1}{15}\log\tfrac{1}{2}$$

$$\therefore \quad \log|\theta - 20| = \frac{t}{15}\log\frac{1}{2} + \log 80 \ldots [\text{From (i)}]$$
When $t = 1$ hour $= 60$ minutes,
$$\log|\theta - 20| = \frac{60}{15}\log\frac{1}{2} + \log 80$$
$$\Rightarrow \log\left(\frac{\theta - 20}{80}\right) = 4\log\frac{1}{2}$$
$$\Rightarrow \frac{\theta - 20}{80} = \left(\frac{1}{2}\right)^4$$
$$\Rightarrow \theta = 5 + 20 = 25^\circ\mathrm{C}$$

Question97

If $x = \sec\theta - \cos\theta, y = \sec^{10}\theta - \cos^{10}\theta$ and $\left(x^2 + 4\right)\left(\frac{dy}{dx}\right)^2 = k\left(y^2 + 4\right)$, then the value of k is MHT CET 2024 (02 May Shift 2)

Options:

A. $\frac{1}{100}$

B. 1

C. 10

D. 100

Answer: D

Solution:

$$x=\sec\theta-\cos\theta \text{ and } y=\sec^{10}\theta-\cos^{10}\theta$$

$$\therefore \quad \frac{dx}{d\theta}=\sec\theta\tan\theta+\sin\theta \text{ and}$$

$$\frac{dy}{d\theta}=10\sec^9\theta\cdot\sec\theta\tan\theta-10\cos^9\theta\cdot(-\sin\theta)$$

$$=10\sec^{10}\theta\tan\theta+10\cos^9\theta\sin\theta$$

$$\therefore \quad \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}=\frac{10\sec^{10}\theta\tan\theta+10\cos^9\theta\sin\theta}{\sec\theta\tan\theta+\sin\theta}=\frac{10\left(\sec^{10}\theta+\cos^{10}\theta\right)}{\sec\theta+\cos\theta}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)^2=\frac{10^2\left(\sec^{10}\theta+\cos^{10}\theta\right)^2}{(\sec\theta+\cos\theta)^2}$$

$$=\frac{100\left[\left(\sec^{10}\theta-\cos^{10}\theta\right)^2+4\sec^{10}\theta\cos^{10}\theta\right]}{(\sec\theta-\cos\theta)^2+4\sec\theta\cos\theta}$$

$$\therefore \quad \left(\frac{dy}{dx}\right)^2=\frac{100\left(y^2+4\right)}{x^2+4}$$

$$\Rightarrow\left(x^2+4\right)\left(\frac{dy}{dx}\right)^2=100\left(y^2+4\right)\Rightarrow \mathrm{k}=100$$

Question98

If $y=y(x)$ is the solution of the differential equation $\left(\frac{5+e^x}{2+y}\right)\frac{dy}{dx}+e^x=0$ satisfying $y(0)=1$, then a value of $y(\log 13)$ is MHT CET 2024 (02 May Shift 2)

Options:

A. -1

B. 0

C. 1

D. 2

Answer: A

Solution:

$$\left(\frac{5+e^x}{2+y}\right)\frac{dy}{dx}+e^x=0$$
$$\Rightarrow \frac{dy}{2+y}=\frac{-e^x}{5+e^x}dx$$
Integrating on both sides, we get

$$\log|2+y|=-\log|5+e^x|+\log|c|$$
$$\Rightarrow \log|2+y|=\log\left|\frac{c}{5+e^x}\right|$$
Since $y(0)=1$ i.e., $y=1$ when $x=0$

$$\therefore \quad \log 3=\log\left|\frac{c}{6}\right|$$
$$\Rightarrow 3=\frac{c}{6}$$
$$\Rightarrow \quad c=18$$
$$\therefore \quad \log|2+y|=\log\left|\frac{18}{5+e^x}\right| \quad \ldots[\text{From (i)}]$$
$$\Rightarrow 2+y=\frac{18}{5+e^x}$$
$$\Rightarrow y=\frac{18}{5+e^x}-2$$

$$\Rightarrow y(\log 13)=\frac{18}{5+e^{\log 13}}-2$$
$$=\frac{18}{5+13}-2$$
$$=-1$$

Question99

If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is equal to MHT CET 2024 (02 May Shift 2)

Options:

A. $-\frac{2}{3}$

B. $-\frac{1}{3}$

C. $\frac{4}{3}$

D. $\frac{1}{3}$

Answer: D

Solution:

$$(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$$

$$\therefore \quad \frac{1}{y+1}\,dy=\frac{-\cos x}{2+\sin x}\,dx$$

Integrating on both sides, we get

$$\log(y+1)=-\log(2+\sin x)+c$$

When $x=0, y=1$

$$\Rightarrow c=2\log 2=\log 4$$

$$\therefore \quad \text{(i)} \Rightarrow \log(y+1)=-\log(2+\sin x)+\log 4$$

$$\Rightarrow \log(y+1)=\log\left(\frac{4}{2+\sin x}\right)$$

$$\Rightarrow y+1=\frac{4}{2+\sin x}$$

$$\Rightarrow y=\frac{4}{2+\sin x}-1$$

$$\therefore \quad y\left(\frac{\pi}{2}\right)=\frac{4}{3}-1=\frac{1}{3}$$

Question100

The solution of the differential equation $\frac{dy}{dx}=(x-y)^2$ when $y(1)=1$ is MHT CET 2024 (02 May Shift 1)

Options:

A. $\log\left|\frac{2-y}{2-x}\right|=2(y-1)$

B. $-\log\left|\frac{1+x-y}{1-x+y}\right|=x+y-2$

C. $\log\left|\frac{2-x}{2-y}\right|=x-y$

D. $-\log\left|\frac{1-x+y}{1+x-y}\right|=2(x-1)$

Answer: D

Solution:

Given differential equation is

$$\frac{dy}{dx} = (x - y)^2$$

Let $x - y = \mathrm{t}$

$$\therefore \quad 1 - \frac{dy}{dx} = \frac{dt}{dx}$$
$$\Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$$

From (i)

$$1 - \frac{dt}{dx} = t^2$$
$$1 - t^2 = \frac{dt}{dx}$$

$$\therefore \quad dx = \frac{1}{1-t^2} dt$$

Integrating on both sides, we get

$$\int dx = \int \frac{1}{1-t^2} dt$$
$$x = \frac{1}{2} \log \left| \frac{1+t}{1-t} \right| + c$$
$$x = \frac{1}{2} \log \left| \frac{1+x-y}{1-(x-y)} \right| + c$$
$$x = \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| + c$$

But $y(1) = 1$

$$\therefore \quad c = 1$$
$$x = \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right| + 1$$
$$x - 1 = \frac{1}{2} \log \left| \frac{1+x-y}{1-x+y} \right|$$
$$2(x-1) = \log \left| \frac{1+x-y}{1-x+y} \right|$$
$$\Rightarrow -\log \left| \frac{1-x+y}{1+x-y} \right| = 2(x-1)$$

Question101

If $x \frac{dy}{dx} = y(\log y - \log x + 1)$, then general solution of this equation is MHT CET 2024 (02 May Shift 1)

Options:

A. $\log\left(\frac{x}{y}\right) = cy$, where c is a constant of integration.

B. $\log\left(\frac{x}{y}\right) = cx$, where c is a constant of integration.

C. $\log\left(\frac{y}{x}\right) = cy$, where c is a constant of integration.

D. $\log\left(\frac{y}{x}\right) = cx$, where c is a constant of integration:

Answer: D

Solution:

$$x\frac{dy}{dx} = y(\log y - \log x + 1).$$
$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\log\left(\frac{y}{x}\right) + 1\right]$$
$$\text{Put } v = \frac{y}{x}$$
$$\therefore \quad y = vx$$
$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$
From (i)

$$\therefore \quad v + x\frac{dv}{dx} = v(\log v + 1)$$
$$v + x\frac{dv}{dx} = v\log v + v$$
$$x\frac{dv}{dx} = v\log v$$
$$\frac{1}{v\log v}dv = \frac{dx}{x}$$
Integrating on both sides, we get

$$\int \frac{1}{v\log v}dv = \int \frac{dx}{x}$$

$$\log(\log v) = \log x + c_1$$
$$\log(\log v) = \log x + \log c \text{ where, } c_1 = \log c$$
$$\Rightarrow \log(\log v) = \log(xc)$$
$$\Rightarrow \log v = xc$$
$$\Rightarrow \log\left(\frac{y}{x}\right) = cx$$

Question102

The differential equation representing the family of curves $y^2 = 2c(x + \sqrt{c})$, where c is a positive parameter, is of MHT CET 2023 (14 May Shift 2)

Options:

A. order 1, degree 4

B. order 2, degree 3

C. order 2, degree 4

D. order 1, degree 3

Answer: D

Solution:

$y^2 = 2c(x + \sqrt{c}) \ldots (i)$

Differentiating w.r.t. x, we get

$2y\frac{dy}{dx} = 2c \ldots (ii)$

Substituting (ii) in (i), we get

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$$
$$\Rightarrow y = 2x\frac{dy}{dx} + 2\frac{dy}{dx}\sqrt{y\frac{dy}{dx}}$$
$$\Rightarrow \left(y - 2x\frac{dy}{dx}\right)^2 = 4y\left(\frac{dy}{dx}\right)^3$$

This is a differential equation of order 1 and degree 3 .

Question103

General solution of the differential equation $\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$ **is MHT CET 2023 (14 May Shift 1)**

Options:

A. $(1 + \cos x)(1 + \sin y) =$ c, where c is a constant of integration.

B. $1 + \sin x + \cos y =$ c, where c is a constant of integration.

C. $(1 + \sin x)(1 + \cos y) =$ c, where c is a constant of integration.

D. $1 + \sin x \cos y =$ c, where c is a constant of integration.

Answer: C

Solution:

$$\cos x(1 + \cos y)dx - \sin y(1 + \sin x)dy = 0$$
$$\Rightarrow \frac{\cos x}{1 + \sin x}dx - \frac{\sin y}{1 + \cos y}dy = 0$$

Integrating on both sides, we get

$$\log|1 + \sin x| + \log|1 + \cos y| = \log|c|$$
$$\Rightarrow \log|(1 + \sin x)(1 + \cos y)| = \log|c|$$
$$\Rightarrow (1 + \sin x)(1 + \cos y) = c$$

Question104

The differential equation of $y=\mathrm{e}^{x}(\mathrm{a}\cos x+\mathrm{b}\sin x)$ is MHT CET 2023 (14 May Shift 1)

Options:

A. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$

B. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2\frac{\mathrm{d}y}{\mathrm{d}x}+2y=0$

C. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}+y=0$

D. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}+2y=0$

Answer: D

Solution:

$$y=\mathrm{e}^{x}(\mathrm{a}\cos x+\mathrm{b}\sin x)$$
$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{x}(\mathrm{a}\cos x+\mathrm{b}\sin x)$$
$$+\mathrm{e}^{x}(\mathrm{b}\cos x-\mathrm{a}\sin x)$$
$$\Rightarrow\frac{\mathrm{d}y}{\mathrm{d}x}=y+\mathrm{e}^{x}(\mathrm{b}\cos x-\mathrm{a}\sin x)$$
$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{e}^{x}(\mathrm{b}\cos x-\mathrm{a}\sin x)$$
$$+\mathrm{e}^{x}(-\mathrm{b}\sin x-\mathrm{a}\cos x)$$
$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}-y\right)-y$$
$$\Rightarrow\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x}+2y=0$$

Question105

The solution of $(1+xy)y\,\mathrm{d}x+(1-xy)x\,\mathrm{d}y=0$ is. MHT CET 2023 (13 May Shift 2)

Options:

A. $\log\left(\frac{x}{y}\right)+\frac{1}{xy}=\mathrm{k}$, where k is constant of integration.

B. $\log\left(\frac{x}{y}\right)=\frac{1}{xy}+\mathrm{k}$, where k is constant of integration.

C. $\log\left(\frac{x}{y}\right)+xy=$ k, where k is constant of integration.

D. $\log\left(\frac{x}{y}\right)=xy+$ k, where k is constant of integration.

Answer: B

Solution:

$(1+xy)y\,\mathrm{d}x+(1-xy)x\,\mathrm{d}y=0$

$\Rightarrow y\,\mathrm{d}x+x\,\mathrm{d}y+xy^2\,\mathrm{d}x-x^2y\,\mathrm{d}y=0$

$\Rightarrow \frac{y\,\mathrm{d}x+x\,\mathrm{d}y}{x^2y^2}+\frac{\mathrm{d}x}{x}-\frac{\mathrm{d}y}{y}=0$

$\Rightarrow \frac{\mathrm{d}(xy)}{x^2y^2}+\frac{\mathrm{d}x}{x}-\frac{\mathrm{d}y}{y}=0$

Integrating on both sides, we get

$-\frac{1}{xy}+\log x-\log y=\mathrm{k}$

$\Rightarrow \log\left(\frac{x}{y}\right)=\frac{1}{xy}+\mathrm{k}$

Question106

If $x\,\mathrm{d}y=y(\mathrm{d}x+y\,\mathrm{d}y), y(1)=1, y(x)>0$, then $y(-3)$ is MHT CET 2023 (13 May Shift 2)

Options:

A. 1

B. 2

C. 3

D. 4

Answer: C

Solution:

$x \, \mathrm{d}y = y(\, \mathrm{d}x + y \, \mathrm{d}y)$

$\Rightarrow y \, \mathrm{d}x = (x - y^2) \, \mathrm{d}y \Rightarrow \frac{\mathrm{d}x}{\mathrm{d}y} + \left(-\frac{1}{y}\right) x = -y$

$\therefore$ I.F. $= \mathrm{e}^{\int -\frac{1}{dy} y^y} = \mathrm{e}^{-\log y} = \frac{1}{y}$

$\therefore$ Solution of the given equation is

$x \cdot \frac{1}{y} = \int -y \cdot \frac{1}{y} \, \mathrm{d}y + \mathrm{c}$

$\Rightarrow \frac{x}{y} = -y + \mathrm{c}$ ....(i)

Since $y(1) = 1$, i.e., $y = 1$ when $x = 1$

$\therefore \quad 1 = -1 + c \Rightarrow c = 2$

$\therefore \quad \frac{x}{y} = -y + 2 \quad \ldots$[From (i)]

Putting $x = -3$, we get

$-\frac{3}{y} = -y + 2$

$\Rightarrow y^2 - 2y - 3 = 0$
$\Rightarrow (y - 3)(y + 1) = 0$

Since $y(x) > 0, y = 3$

Question107

The parametric equations of the curve $x^2 + y^2 + ax + by = 0$ are MHT CET 2023 (13 May Shift 1)

Options:

A. $x = \frac{\mathrm{a}}{2} + \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \cos\theta, y = \frac{\mathrm{b}}{2} + \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \sin\theta$

B. $x = \frac{\mathrm{a}}{2} - \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \cos\theta, y = \frac{\mathrm{b}}{2} - \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \sin\theta$

C. $x = -\frac{\mathrm{a}}{2} + \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \cos\theta, y = -\frac{\mathrm{b}}{2} + \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \sin\theta$

D. $x = -\frac{\mathrm{a}}{2} - \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \cos\theta, y = -\frac{\mathrm{b}}{2} - \sqrt{\frac{\mathrm{a}^2+\mathrm{b}^2}{4}} \sin\theta$

Answer: C

Solution:

$$x^2+y^2+ax+by=0$$
$$\Rightarrow\left(x+\frac{a}{2}\right)^2+\left(y+\frac{b}{2}\right)^2=\frac{a^2+b^2}{4}$$

Comparing with $(x-h)^2+(y-k)^2=r^2$, we get

$$h=-\frac{a}{2},k=-\frac{b}{2},r=\sqrt{\frac{a^2+b^2}{4}}$$

$\therefore$ The parametric equations are

$$x=-\frac{a}{2}+\sqrt{\frac{a^2+b^2}{4}}\cos\theta, y=-\frac{b}{2}+\sqrt{\frac{a^2+b^2}{4}}\sin\theta$$

Question108

The solution of the differential equation $e^{-x}(y+1)dy+\left(\cos^2 x-\sin 2x\right)y\,dx=0$ at $x=0, y=1$ is MHT CET 2023 (13 May Shift 1)

Options:

A. $(y+1)+e^x\cos^2 x=2$

B. $y+\log y=e^x\cos^2 x$

C. $\log(y+1)+e^x\cos^2 x=1$

D. $y+\log y+e^x\cos^2 x=2$

Answer: D

Solution:

$$e^{-x}(y+1)dy+\left(\cos^2 x-\sin 2x\right)y\,dx=0$$
$$\Rightarrow\frac{y+1}{y}dy+e^x\left(\cos^2 x-\sin 2x\right)dx=0$$

Integrating on both sides, we get

$$\int\left(1+\frac{1}{y}\right) \mathrm{d}y+\int \mathrm{e}^x\left(\cos^2 x-2 \sin x \cos x\right) \mathrm{d}x=\mathrm{c}$$

$$\Rightarrow y+\log |y|+\mathrm{e}^x \cos^2 x=\mathrm{c}$$

At $x=0, y=1$

$$\therefore \quad 1+\log 1+\mathrm{e}^0 \cos^2 0=\mathrm{c}$$

$$\Rightarrow \mathrm{c}=2$$

$$\therefore \quad y+\log |y|+\mathrm{e}^x \cos^2 x=2$$

Question109

The particular solution of differential equation $\mathrm{e}^{\frac{dy}{dx}}=(x+1), y(0)=3$ is MHT CET 2023 (13 May Shift 1)

Options:

A. $y=x \log x-x+2$

B. $y=(x+1) \log (x+1)-x+3$

C. $y=(x+1) \log (x+1)+x-3$

D. $y=x \log x+x-2$

Answer: B

Solution:

$$\mathrm{e}^{\frac{\mathrm{d}}{\mathrm{d}x}}=(x+1)$$

$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\log (x+1)$$

Integrating on both sides, we get

$$\int \mathrm{d}y=\int \log (x+1) \mathrm{d}x+\mathrm{c}$$

$$\begin{aligned}
\Rightarrow y & =x \log (x+1)-\int \frac{x}{x+1} \mathrm{d}x+\mathrm{c} \\
& =x \log (x+1)-\int \frac{x+1-1}{x+1} \mathrm{d}x+\mathrm{c} \\
& =x \log (x+1)-\int\left(1-\frac{1}{x+1}\right) \mathrm{d}x+\mathrm{c} \\
y & =x \log (x+1)-x+\log (x+1)+\mathrm{c} .
\end{aligned}$$

Since $y(0)=3$, i.e., $y=3$ when $x=0$

$$3=0+c \Rightarrow c=3$$
$$y=x\log(x+1)+\log(x+1)-x+3$$
$$y=(x+1)\log(x+1)-x+3$$

Question110

The solution of $e^{y-x}\frac{dy}{dx}=\frac{y(\sin x+\cos x)}{(1+y\log y)}$ is MHT CET 2023 (12 May Shift 2)

Options:

A. $\frac{e^y}{y}=e^x\sin x+c$, where c is a constant of integration.

B. $e^y\log y=e^x\cos x+c$, where c is a constant of integration.

C. $e^y\log y=e^x\sin x+c$, where c is a constant of integration.

D. $e^y y=e^x\sin x+c$, where c is a constant of integration.

Answer: C

Solution:

$$e^{y-x}\frac{dy}{dx}=\frac{y(\sin x+\cos x)}{(1+y\log y)}$$
$$\therefore \quad e^y\frac{(1+y\log y)}{y}dy=e^x(\sin x+\cos x)dx$$
$$\therefore \quad \int e^y\left(\log y+\frac{1}{y}\right)dy=\int e^x(\sin x+\cos x)dx$$
$$\Rightarrow e^y\log y=e^x\sin x+c$$
$$\ldots\left[\because \int e^x\left(f(x)+f'(x)\right)dx=e^x f(x)+c\right]$$

Question111

If $(2+\sin x)\frac{dy}{dx}+(y+1)\cos x=0$ and $y(0)=1$, then $y\left(\frac{\pi}{2}\right)$ is MHT CET 2023 (12 May Shift 2)

Options:

A. $\frac{-2}{3}$

B. $\frac{-1}{3}$

C. $\frac{4}{3}$

D. $\frac{1}{3}$

Answer: D

Solution:

$$(2+\sin x)\frac{\mathrm{d}y}{\mathrm{d}x}+(y+1)\cos x=0$$

$\therefore \quad \frac{1}{y+1}\,\mathrm{d}y=\frac{-\cos x}{2+\sin x}\,\mathrm{d}x$

$\therefore \quad$ Integrating both sides, we get

$$\log(y+1)=-\log(2+\sin x)+\mathrm{c}\ldots\text{ (i)}$$

when $x=0, y=1$

$$\Rightarrow \mathrm{c}=2\log 2$$

$\therefore \quad \text{(i)} \Rightarrow \log(y+1)=-\log(2+\sin x)+\log 4$

$$\Rightarrow \log(y+1)=\log\left(\frac{4}{2+\sin x}\right)$$

$$\Rightarrow y+1=\frac{4}{2+\sin x}$$

When $x=\frac{\pi}{2}$, (ii) $\Rightarrow y=\frac{4}{3}-1=\frac{1}{3}$

Question112

The differential equation $\cos(x+y)\mathrm{d}y=\mathrm{d}x$ has the general solution given by MHT CET 2023 (12 May Shift 1)

Options:

A. $y=\sin(x+y)+\mathrm{c}$, where c is a constant.

B. $y=\tan(x+y)+\mathrm{c}$, where c is a constant

C. $y=\tan\left(\frac{x+y}{2}\right)+\mathrm{c}$, where c is a constant

D. $y=\frac{1}{2}\tan(x+y)+\mathrm{c}$, where c is a constant

Answer: C

Solution:

$$\cos(x+y)\mathrm{d}y=\mathrm{d}x$$

$$\therefore \quad \frac{\mathrm{d}x}{\mathrm{d}y}=\cos(x+y)$$

Put $x+y=\mathrm{u}$ Differentiating w.r.t. y, we get $\frac{\mathrm{d}x}{\mathrm{d}y}+1=\frac{\mathrm{du}}{\mathrm{d}y}$

$$\therefore \quad \frac{dx}{dy}=\frac{du}{dy}-1$$

Substituting (ii) and (iii) in (i), we get

$$\frac{du}{dy}-1=\cos u$$
$$\therefore \quad \frac{du}{1+\cos u}=dy$$
$$\therefore \quad \frac{du}{2\cos^2\left(\frac{u}{2}\right)}=dy$$

Integrating on both sides, we get

$$\frac{1}{2}\int \sec^2\left(\frac{u}{2}\right)du=\int dy$$
$$\therefore \quad y=\tan\left(\frac{x+y}{2}\right)+c$$

Question113

If $\frac{dy}{dx}=y+3$ and $y(0)=2$, then $y(\log 2)=$ MHT CET 2023 (12 May Shift 1)

Options:

A. 5

B. 7

C. 13

D. -2

Answer: B

Solution:

$$\frac{dy}{dx}=y+3$$
$$\Rightarrow \frac{dy}{y+3}=dx$$

Integrating on both sides, we get

$$\int \frac{dy}{y+3} = \int dx + c$$

$$\Rightarrow \quad \log(y+3) = x + c$$

$$y = 2 \text{ when } x = 0$$

$$\therefore \quad \log(2+3) = 0 + c \Rightarrow c = \log 5$$

$$\therefore \quad \log(y+3) = x + \log 5$$

$$\Rightarrow y + 3 = 5e^{x}$$

$$\Rightarrow y = 5e^{x} - 3$$

$$\therefore \quad y(\log 2) = 5e^{\log 2} - 3 = 10 - 3$$

$$= 7$$

Question114

If $\log(x+y) = 2xy$, then $\frac{dy}{dx}$ at $x = 0$ is MHT CET 2023 (12 May Shift 1)

Options:

A. 1

B. -1

C. 2

D. -2

Answer: A

Solution:

$$\log(x+y) = 2xy$$

Differentiating w.r.t. x, we get

$$\frac{1}{x+y}\left(1 + \frac{dy}{dx}\right) = 2y + 2x\frac{dy}{dx}$$

$$\therefore \quad \frac{1}{x+y} + \frac{1}{x+y}\frac{dy}{dx} = 2y + 2x\frac{dy}{dx}$$

$$\text{At } x = 0, \text{(i)} \Rightarrow y = 1$$

$$\therefore \quad \text{(ii)} \Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = 1$$

Question115

The solution of $\frac{dx}{dy} + \frac{x}{y} = x^2$ is MHT CET 2023 (11 May Shift 2)

Options:

A. $\frac{1}{y} = cx - x \log x$, where c is a constant of integration.

B. $\frac{1}{x} = cy - y \log y$, where c is a constant of integration.

C. $\frac{1}{x} = cx - x \log y$, where c is a constant of integration.

D. $\frac{1}{y} = cx - y \log x$, where c is a constant of integration.

Answer: B

Solution:

$\frac{dx}{dy} + \frac{x}{y} = x^2$

$\therefore \quad \frac{1}{x^2}\frac{dx}{dy} + \frac{1}{xy} = 1..(i)$

Let $\frac{1}{x} = t$

Differentiating w.r.t. y, we get

$$\frac{-1}{x^2}\frac{dx}{dy} = \frac{dt}{dy} \Rightarrow \frac{1}{x^2}\frac{dx}{dy} = \frac{-dt}{dy}$$

$$\therefore \quad \text{(i)} \Rightarrow \frac{-dt}{dy} + \frac{t}{y} = 1$$

$$\therefore \quad \frac{dt}{dy} - \frac{t}{y} = -1$$

$$\therefore \quad \text{I.F.} = e^{\int \frac{-1}{y} d} = e^{\log\left(\frac{1}{y}\right)} = \frac{1}{y}$$

$\therefore$ The solution of the given equation is

$$t(\text{I.F}) = \int (-1)(\text{I.F.})dy + c$$

$$t\left(\frac{1}{y}\right) = \int \frac{-1}{y} dy + c$$

$$\therefore \quad \frac{t}{y} = -\log y + c$$

$$\therefore \quad \frac{1}{xy} = -\log y + c$$

$$\therefore \quad \frac{1}{x} = cy - y \log y$$

Question116

The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = \sin x$ is MHT CET 2023 (11 May Shift 2)

Options:

A. $xy + \cos x = \sin x + c$, where c is a constant of integration.

B. $x(y+\cos x)=\sin x+\mathrm{c}$, where c is a constant of integration.

C. $y(x+\cos x)=\sin x+\mathrm{c}$, where c is a constant of integration.

D. $xy+\sin x=\cos x+c$, where c is a constant of integration.

Answer: B

Solution:

For given linear differential equation,

$$\text{I.F.} = \mathrm{e}^{\int \frac{1}{x} dx} = \mathrm{e}^{\log x} = x$$

$\therefore$ The required solution is

$$yx = \int x \sin x \frac{\mathrm{d}y}{\mathrm{d}x}$$

$$\therefore \quad yx = -x\cos x + \int \cos x \, \mathrm{d}x$$
$$\therefore \quad yx = -x\cos x + \sin x + \mathrm{c}$$
$$\therefore \quad x(y+\cos x) = \sin x + \mathrm{c}$$

Question117

The solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1+y^2}{1+x^2}$ is MHT CET 2023 (11 May Shift 1)

Options:

A. $x+y=\mathrm{c}$, where c is a constant of integration.

B. $x-y=\mathrm{c}(xy)$, where c is a constant of integration.

C. $x+y=\mathrm{c}(1+xy)$, where c is a constant of integration.

D. $y-x=\mathrm{c}(1+xy)$, where c is a constant of integration.

Answer: D

Solution:

$$\boxed{y - x = c(1 + xy)}$$

Solution

Given

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}.$$

This is separable:

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}.$$

Integrate:

$$\tan^{-1} y = \tan^{-1} x + C.$$

Let $C = \tan^{-1} k$ (so $k = \tan C$). Then using $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$,

$$y = \tan(\tan^{-1} x + \tan^{-1} k) = \frac{x+k}{1-kx}.$$

Rearrange:

$$y(1-kx) = x + k \;\Rightarrow\; y - x = k(1+xy).$$

Rename k as the constant c:

$$\boxed{y - x = c(1+xy)}.$$

Question118

The general solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + \left(\frac{3x^2}{1+x^3}\right) y = \frac{1}{x^3+1}$ is MHT CET 2023 (10 May Shift 2)

Options:

A. $y\left(1+x^3\right) = x^3 + \mathrm{c}$, where c is a constant of integration.

B. $y\left(1+x^3\right) = x + \mathrm{c}$, where c is a constant of integration.

C. $y\left(1+x^3\right) = x^2 + \mathrm{c}$, where c is a constant of integration.

D. $y\left(1+x^3\right) = 2x + \mathrm{c}$, where c is a constant of integration.

Answer: B

Solution:

Given differential equation is

$$\frac{dy}{dx}+\left(\frac{3x^2}{1+x^3}\right)y=\frac{1}{x^3+1}$$

Here, $P=\frac{3x^2}{1+x^3}, Q=\frac{1}{x^3+1}$

$\therefore$ I.F. $=e^{\int\frac{3x^2}{1+x^3}dx}=e^{\log(1+x^3)}=(1+x^3)$

$\therefore$ Solution of the given equation is

$y(1+x^3)=\int\frac{1}{1+x^3}\cdot(1+x^3)\,dx+c$

$\Rightarrow y(1+x^3)=x+c$

Question119

In a certain culture of bacteria, the rate of increase is proportional to the number of bacteria present at that instant. It is found that there are 10,000 bacteria at the end of 3 hours and 40,000 bacteria at the end of 5 hours, then the number of bacteria present in the beginning are MHT CET 2023 (10 May Shift 2)

Options:

A. 1250.

B. 1200.

C. 1350.

D. 1300.

Answer: A

Solution:

Let x be the number of bacteria present at time t.

$$\therefore \quad \frac{\mathrm{d}x}{\mathrm{dt}} \propto x$$
$$\therefore \quad \frac{\mathrm{d}x}{\mathrm{dt}} = \mathrm{k}x$$
$$\therefore \quad \frac{\mathrm{d}x}{x} = \mathrm{kdt}$$

Integrating on both sides, we get

$\log x = \mathrm{kt} + \mathrm{c} \ldots (i)$

When $\mathrm{t} = 3, x = 10,000$

Equation (i) becomes

$\log(10,000) = 3\mathrm{k} + \mathrm{c} \ldots (ii)$

When $\mathrm{t} = 5, x = 40,000$

Equation (i) becomes

$\log(40,000) = 5\mathrm{k} + \mathrm{c} \ldots (iii)$

Subtracting (ii) from (iii), we get

$\mathrm{k} = \log 2$

From equation (ii),

$\log(10,000) = 3\log 2 + c$

$\therefore \quad c = \log(1250)$

Now, Initially $\mathrm{t} = 0$

From (i),

$\log x = \mathrm{k} \times 0 + \log(1250)$

$\therefore \quad \log x = \log 1250$

$\therefore \quad x = 1250$

Question120

The differential equation of all circles, passing through the origin and having their centres on the X-axis, is MHT CET 2023 (10 May Shift 2)

Options:

A. $y^2 = x^2 + xy\frac{dy}{dx}$

B. $x^2 = y^2 + 2xy\frac{dy}{dx}$

C. $y^2 = x^2 + 2xy\frac{dy}{dx}$

D. $x^2 = y^2 - xy\frac{dy}{dx}$

Answer: C

Solution:

The system of circles which passes through origin and whose centre lies on X-axis is
$x^2 + y^2 - 2bx = 0 \ldots (i)$

Differentiating w.r.t x, we get

$2x + 2y\frac{dy}{dx} = 2\text{ b} \ldots (ii)$

Substituting (ii) in (i), we get

$$x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$$
$$\Rightarrow y^2 - x^2 - 2xy\frac{dy}{dx} = 0$$
$$\Rightarrow y^2 = x^2 + 2xy\frac{dy}{dx}$$

Question121

The population $P = P(t)$ at time t of certain species follows the differential equation $\frac{dP}{dt} = 0.5P - 450$. If $P(0) = 850$, then the time at which population becomes zero is MHT CET 2023 (10 May Shift 1)

Options:

A. $2\log 18$

B. $\log 9$

C. $\frac{1}{2}\log 18$

D. $\log 18$

Answer: A

Solution:

$$\frac{dP}{dt} = 0.5P - 450$$
$$= \frac{P}{2} - \frac{900}{2}$$
$$\therefore \quad \frac{dP}{dt} = \frac{P - 900}{2}$$
$$\therefore \quad \frac{2dP}{P - 900} = dt$$

Integrating on both sides, we get

Given differential equation is $2 \log |P - 900| = t + c$

$P(0) = 850$ i.e., $P = 850$ when $t = 0$

$c = 2 \log 50$

$2 \log |P - 900| = t + 2 \log 50$

When $P = 0$,

$2 \log 900 = t + 2 \log 50$

$\Rightarrow t = 2(\log 900 - \log 50)$

$$= 2 \log \frac{900}{50} = 2 \log 18$$

Question122

The differential equation $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$ determines a family of circles with MHT CET 2023 (10 May Shift 1)

Options:

A. variable radii and fixed centre at $(0, 1)$.

B. variable radii and fixed centre at $(0, -1)$.

C. fixed radius of 1 unit and variable centre along the Y-axis.

D. fixed radius of 1 unit and variable centre along the X-axis.

Answer: D

Solution:

$$\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{y}$$

$\therefore \quad \int \frac{y}{\sqrt{1-y^2}} \, dy = \int 1 \, dx$

$\therefore \quad -\sqrt{1-y^2} = x + c$

$\therefore \quad (x+c)^2 = 1 - y^2$

$\therefore \quad (x+c)^2 + y^2 = 1$

$\therefore$ Radius is fixed, which is 1 and the centre is $(-c, 0)$ which is a variable centre on the X-axis.

Question123

General solution of the differential equation $\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$ is MHT CET 2023 (10 May Shift 1)

Options:

A. $(1+\cos x)(1+\sin y) = c$

B. $1 + \sin x + \cos y = c$

C. $(1+\sin x)(1+\cos y) = c$

D. $1 + \sin x \cdot \cos y = c$

Answer: C

Solution:

Given differential equation is

$$\cos x(1+\cos y)dx - \sin y(1+\sin x)dy = 0$$
$$\Rightarrow \cos x(1+\cos y)dx = \sin y(1+\sin x)dy$$
$$\Rightarrow \frac{\cos x}{1+\sin x} dx = \frac{\sin y}{1+\cos y} dy$$

Integrating on both sides, we get

$$\int \frac{\cos x}{1+\sin x} dx = \frac{\sin y}{1+\cos y} dy + \log|c|$$
$$\Rightarrow \log|1+\sin x| = -\log|1+\cos y| + \log|c|$$
$$\Rightarrow \log|1+\sin x| + \log|1+\cos y| = \log|c|$$
$$\Rightarrow \log|(1+\sin x)(1+\cos y)| = \log|c|$$
$$\Rightarrow (1+\sin x)(1+\cos y) = c$$

Question124

If $f'(x) = x - \frac{5}{x^5}$ and $f(1) = 4$, then $f(x)$ is MHT CET 2023 (09 May Shift 2)

Options:

A. $\frac{x^2}{2}+\frac{9}{4}\frac{1}{x^4}+\frac{5}{4}$

B. $\frac{x^2}{2}-\frac{5}{4}\frac{1}{x^4}+\frac{9}{4}$

C. $\frac{x^2}{2}+\frac{5}{4}\frac{1}{x^4}+\frac{9}{4}$

D. $\frac{x^2}{2}-\frac{9}{4}\frac{1}{x^4}+\frac{5}{4}$

Answer: C

Solution:

Given $f'(x)=x-\frac{5}{x^5}$

$\therefore$ Integrating both sides, we get

$$f(x)=\int\left(x-\frac{5}{x^5}\right)dx$$
$$f(x)=\frac{x^2}{2}+\frac{5}{4}\times\frac{1}{x^4}+c$$

$\therefore$ But $f(1)=4$

$\therefore$ $\frac{1}{2}+\frac{5}{4}+c=4$

$\therefore$ $c=\frac{9}{4}$

$\therefore$ $f(x)=\frac{x^2}{2}+\frac{5}{4}\frac{1}{x^4}+\frac{9}{4}$

Question125

General solution of the differential equation $\log\left(\frac{dy}{dx}\right)=ax+by$ is MHT CET 2023 (09 May Shift 2)

Options:

A. $ae^{by}+be^{ax}=c_1$, where c_1 is a constant.

B. $ae^{-by}+b^{-ax}=c_1$, where c_1 is a constant.

C. $ae^{-by}+be^{ax}=c_1$, where c_1 is a constant.

D. $ae^{by}+be^{-ax}=c_1$, where c_1 is a constant.

Answer: C

Solution:

Given differential equation is

$$\log\left(\frac{dy}{dx}\right) = ax + by$$

$\therefore \quad \frac{dy}{dx} = e^{ax+by}$

$\therefore \quad \frac{dy}{dx} = e^{ax} \cdot e^{by}$

$\therefore \quad \frac{dy}{e^{by}} = e^{ax} \cdot dx$

$e^{-by}dy - e^{ax}dx = 0$

Integrating both sides, we get

$\int e^{-by}dy - \int e^{ax}dx = 0$

$\frac{e^{-by}}{-b} - \frac{e^{ax}}{a} + c = 0$

i.e., $\frac{e^{-by}}{b} + \frac{e^{xx}}{a} = c$

$ae^{-by} + be^{ax} = abc$

$ae^{-by} + be^{ax} = c_1$, where $c_1 = abc$

Question126

A water tank has a shape of inverted right circular cone whose semi-vertical angle is $\tan^{-1}\left(\frac{1}{2}\right)$. Water is poured into it at constant rate of 5 cubic meter/minute. The rate in meter/minute at which level of water is rising. at the instant when depth of water in the tank is. $10 \mathrm{~m}$ is MHT CET 2023 (09 May Shift 2)

Options:

A. $\frac{1}{5\pi}$

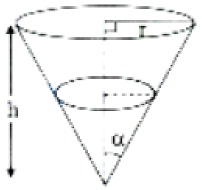
B. $\frac{1}{15\pi}$

C. $\frac{2}{\pi}$

D. $\frac{1}{10\pi}$

Answer: A

Solution:



Semi-vertical angle $= \tan^{-1}\left(\frac{1}{2}\right)$

Let $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$

$\tan\alpha = \frac{1}{2}$

$\frac{\mathrm{r}}{\mathrm{h}} = \frac{1}{2}$

$\mathrm{r} = \frac{\mathrm{h}}{2}$

Given, $\frac{d\mathrm{V}}{\mathrm{dt}} = 5\ \mathrm{m}^3/\mathrm{min}$.

$\mathrm{V} =$ Volume of cone

Volume of cone $= \frac{1}{3}\pi r^2\ \mathrm{h}$

$\mathrm{V} = \frac{1}{3}\pi\left(\frac{\mathrm{h}}{2}\right)^2 \times \mathrm{h}$

$\mathrm{V} = \frac{1}{12}\pi\mathrm{h}^3$

Differentiating w, r.t. t, we get

$\frac{\mathrm{dV}}{\mathrm{dt}} = \frac{1}{12} \times \pi \times 3\ \mathrm{h}^2 \times \frac{\mathrm{dh}}{\mathrm{dt}}$

$5 = \frac{1}{4}\pi h^2\frac{\mathrm{dh}}{\mathrm{dt}}$

$\frac{\mathrm{dh}}{\mathrm{dt}} = \frac{20}{\pi\mathrm{h}^2}$

Now, $\mathrm{h} = 10$... [Given]

$\therefore \quad \frac{\mathrm{dh}}{\mathrm{dt}} = \frac{20}{\pi\times(10)^2}$

$\frac{\mathrm{dh}}{\mathrm{dt}} = \frac{1}{5\pi}$

$\therefore$ Rate of change of water level is $\frac{1}{5\pi}\mathrm{m}/\mathrm{min}$.

Question127

The differential equation of all circles which pass through the origin and whose centres lie on Y-axis is MHT CET 2023 (09 May Shift 2)

Options:

A. $\left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$

B. $\left(x^2-y^2\right)\frac{dy}{dx}+2xy=0$

C. $\left(x^2-y^2\right)\frac{dy}{dx}+xy=0$

D. $\left(x^2-y^2\right)\frac{dy}{dx}-xy=0$

Answer: A

Solution:

Circle passes through origin and centre lie on Y-axis.

Let $(0, k)$ be centre and ' k ' be radius

$\therefore$ Equation of circle is

$$(x-0)^2+(y-\mathrm{k})^2=\mathrm{k}^2$$
$$x^2+y^2-2y\mathrm{k}+\mathrm{k}^2=\mathrm{k}^2$$
$$x^2+y^2-2\mathrm{k}y=0$$
$$x^2+y^2=2\mathrm{k}y\ldots(i)$$
$$\frac{x^2+y^2}{2y}=\mathrm{k}\ldots(ii)$$

Differentiating equation (i) with respect to x, we get

$$2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}=2\mathrm{k}\frac{\mathrm{d}y}{\mathrm{d}x}$$
$$2x+2y\frac{\mathrm{d}y}{\mathrm{d}x}-2\mathrm{k}\frac{\mathrm{d}y}{\mathrm{d}x}=0$$
$$2x+2(y-\mathrm{k})\frac{\mathrm{d}y}{\mathrm{d}x}=0$$
$$2x+2\left[y-\left(\frac{x^2+y^2}{2y}\right)\right]\frac{\mathrm{d}y}{\mathrm{d}x}=0\ldots[From(ii)]$$
$$2x+2\left[\frac{2y^2-x^2-y^2}{2y}\right]\frac{\mathrm{d}y}{\mathrm{d}x}=0$$
$$2x+\left(\frac{y^2-x^2}{y}\right)\frac{\mathrm{d}y}{\mathrm{d}x}=0$$
$$2xy+\left(y^2-x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=0$$
$$\text{i.e. }\left(x^2-y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}-2xy=0$$

Question128

If $x^k+y^k=a^k(a,k>0)$ and $\frac{dy}{dx}+\left(\frac{y}{x}\right)^{\frac{1}{b}}-0$, then k has the value MHT CET 2023 (09 May Shift 2)

Options:

A. $\frac{1}{3}$

B. $\frac{2}{3}$

C. $\frac{1}{4}$

D. $\frac{2}{7}$

Answer: B

Solution:

$x^{k} + y^{k} = a^{k}$

Differentiating w.r.t. x, we get

$$\begin{aligned} & kx^{k-1} + ky^{k-1}\frac{dy}{dx} = 0 \\ \therefore\ & \frac{dy}{dx} = -\frac{kx^{k-1}}{ky^{k-1}} \\ \therefore\ & \frac{dy}{dx} = -\left(\frac{x}{y}\right)^{k-1} \\ & \frac{dy}{dx} + \left(\frac{y}{x}\right)^{1-k} = 0 \\ & \text{But } \frac{dy}{dx} + \left(\frac{y}{x}\right)^{\frac{1}{3}} = 0 \ldots [Given] \end{aligned}$$

Comparing above equations, we get

$$\begin{aligned} 1 - k &= \frac{1}{3} \\ 1 - \frac{1}{3} &= k \\ \therefore\ k &= \frac{2}{3} \end{aligned}$$

Question129

The differential equation of all parabolas, whose axes are parallel to Y-axis, is MHT CET 2023 (09 May Shift 1)

Options:

A. $y_3 = 1$

B. $y_3 = -1$

C. $y_3 = 0$

D. $yy_3 + y_1 = 0$

Answer: C

Solution:

Parabola whose axes are parallel to Y-axis. Vertex is not $(0,0)$

Equation becomes

$(x-\mathrm{h})^2=4\,\mathrm{b}(y-\mathrm{k})$

Differentiating w.r.t. x, we get

$2(x-h)=4b\left(\frac{dy}{dx}\right)$

Again differentiating w.r.t. x, we get

$2=4\,\mathrm{b}\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)$

Again differentiating w.r.t. x, we get

$$0=4b\left(\frac{\mathrm{d}^3y}{\mathrm{d}x^3}\right)$$
$$\therefore \quad \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=0$$
$$\text{i.e., } y_3=0$$

Question130

The particular solution of the differential equation $\left(1+y^2\right)\mathrm{d}x-xy\,\mathrm{d}y=0$ at $x=1, y=0$, represents MHT CET 2023 (09 May Shift 1)

Options:

A. circle

B. pair of straight lines

C. hyperbola

D. ellipse

Answer: C

Solution:

$$(1+y^2)\,dx - xy\,dy = 0$$
$$\therefore \quad (1+y^2)\,dx = xy\,dy$$
$$\therefore \quad \frac{1}{x}\,dx = \frac{y\,dy}{1+y^2}$$

Integrating both sides, we get

$$\int \frac{1}{x}\,dx = \int \frac{y}{1+y^2}\,dy$$
$$\log x = \frac{1}{2}\log(1+y^2) + c$$

At $x = 1, y = 0 \ldots [Given]$

$$\therefore \quad 0 = \frac{1}{2}\log(1+0) + c$$
$$\therefore \quad c = 0$$
$$\therefore \quad \log x = \frac{1}{2}\log(1+y^2)$$
$$\therefore \quad x^2 = 1 + y^2$$
$$\therefore \quad x^2 - y^2 = 1,$$

Which is a rectangular hyperbola.

Question131

A spherical raindrop evaporates at a rate proportional to its surface area. If originally its radius is 3 mm and 1 hour later it reduces to 2 mm, then the expression for the radius R of the raindrop at any time t is MHT CET 2023 (09 May Shift 1)

Options:

A. $6R = t + 2$

B. $R(t+2) = 6$

C. $R = 6(t+2)$

D. $6R = t$

Answer: B

Solution:

According to the given conditions, when $t = 0, R = 3$ and when $t = 1, R = 2$

This condition is satisfied by only option (B)

Question132

If $(x^2+y^2)\,dy = xy\,dx$, with $y(x_0) = e, y(1) = 1$, then x_0 has the value MHT CET 2022 (11 Aug Shift 1)

Options:

A. $\sqrt{3}e$

B. $\sqrt{3}e$

C. e

D. $\sqrt{3}e^2$

Answer: A

Solution:

$$\begin{aligned}
&\left(x^2+y^2\right) \mathrm{d}y = xy\, \mathrm{d}x \quad [\text{Let } y = vx] \\
&\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{xy}{x^2+y^2} \\
&\Rightarrow v + x\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v}{1+v} \\
&\Rightarrow -\int \frac{1+v^2}{v^3}\, \mathrm{d}v = \int \frac{\mathrm{d}x}{x} \\
&\Rightarrow \frac{1}{2v^2} - \log v = \log x + \log c \\
&\Rightarrow cy = e^{\frac{x^2}{2y^2}}
\end{aligned}$$

[Let $y = vx$]

Putting $x = 1$ and $y = 1$ we get $c = e^{-\frac{1}{2}}$

$\Rightarrow y = e^{\frac{x^2-y^2}{2y^2}}$, now putting $x = x_0$ and $y = e$ we get $x_0 = \sqrt{3}e$

Question133

If a body is heated to $110^{\circ}\mathrm{C}$ and placed in air at $10^{\circ}\mathrm{C}$ after 1 hour its temperature is $60^{\circ}\mathrm{C}$, then the additional time required for it to cool to $30^{\circ}\mathrm{C}$ is MHT CET 2022 (11 Aug Shift 1)

Options:

A. $\frac{\log 5}{\log 2}$hrs

B. $\frac{\log 2}{\log 5}$hrs

C. $\left(\frac{\log 5}{\log 2} - 1\right)$ hrs

D. $\left(\frac{\log 5}{\log 2} + 1\right)$ hrs

Answer: C

Solution:

$$\frac{dT}{dt} = -K(T-10)$$
$$\Rightarrow T - 10 = e^{-kt+C}$$
$$\Rightarrow T = 10 + e^{C} \cdot e^{-kt}$$

For $t = 0$,

$$T = 110$$
$$\Rightarrow e^{C} = 100 \text{ i.e.}$$
$$T = 10 + 100 \cdot e^{-kt}$$

For $t = 1$,

$$T = 60$$
$$\Rightarrow 60 = 10 + 100 \cdot e^{-k \times 1}$$
$$\Rightarrow -k = \log \frac{1}{2}$$
$$\Rightarrow k = \log 2$$

$$\Rightarrow T = 10 + 100 \cdot e^{-(\log_2)t}$$

Putting $T = 30$

$$30 = 10 + 100 \cdot e^{-(\log_2)t}$$
$$\Rightarrow \log\left(\frac{1}{5}\right) = -(\log 2)t$$
$$\Rightarrow t = \frac{\log 5}{\log 2}$$

Additional time $= t - 1 = \frac{\log 5}{log2} - 1$

Question134

The particular solution of $\frac{y}{x}\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ when $x = 2, y = 1$ is MHT CET 2022 (10 Aug Shift 2)

Options:

A. $\left(1+y^2\right) = 2\left(1+x^2\right)$

B. $2\left(1+y^2\right) = 5\left(1+x^2\right)$

C. $2\left(1+y^2\right) = \left(1+x^2\right)$

D. $5\left(1+y^2\right) = 2\left(1+x^2\right)$

Answer: D

Solution:

$$\frac{y}{x} \cdot \frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$
$$\Rightarrow \frac{y}{1+y^2} dy = \frac{x}{1+x^2} dx$$
$$\Rightarrow \frac{1}{2} \int \frac{2y\, dy}{1+y^2} = \frac{1}{2} \int \frac{2x}{1+x^2} dx$$
$$\Rightarrow \frac{1}{2} \log|1+y^2| = \frac{1}{2} \log|1+x^2| + \frac{1}{2} \log c$$
$$\Rightarrow \log \frac{1+y^2}{1+x^2} = \log c$$
$$\Rightarrow \frac{1+y^2}{1+x^2} = C$$

Putting $x = 2, y = 1$ we get $C = \frac{2}{5}$

$$\Rightarrow \frac{1+y^2}{1+x^2} = \frac{2}{5}$$
$$\Rightarrow 5\left(1+y^2\right) = 2\left(1+x^2\right)$$

Question135

The solution of the differential equation $(1+x)y\, dx + (1-y)x\, dy = 0$ is MHT CET 2022 (10 Aug Shift 2)

Options:

A. $\log xy - x + y = C$

B. $\log\left(\frac{x}{y}\right) - x + y = C$

C. $\log xy - x - y = C$

D. $\log(xy) + x - y = C$

Answer: D

Solution:

$$(1+x)y\, dx + (1-y)x\, dy = 0$$
$$\Rightarrow \int \frac{1+x}{x} dx = \int \frac{y-1}{y} dy$$
$$\Rightarrow \log|x| + x = y - \log|y| + C$$
$$\Rightarrow \log(xy) + x - y = C$$

Question136

The order and degree of a differential equation obtained by eliminating arbitrary constant C from the family of curves $y^2 = 2C(x + \sqrt{C})$ are respectively MHT CET 2022 (10 Aug Shift 1)

Options:

A. 1,3

B. 1,4

C. 1,1

D. 1,2

Answer: A

Solution:

$$y^2 = 2c(x + \sqrt{c}) \quad \ldots\ldots \text{(i)}$$

$$2y\frac{dy}{dx} = 2c$$

$$\Rightarrow c = y \cdot \frac{dy}{dx} \quad \ldots\ldots \text{(ii)}$$

from (i) and (ii)

$$y^2 = 2y\frac{dy}{dx}\left(x + \sqrt{y\frac{dy}{dx}}\right)$$

$$\Rightarrow y - 2x\frac{dy}{dx} = 2\frac{dy}{dx} \cdot \sqrt{y \cdot \frac{dy}{dx}}$$

$$\Rightarrow \left(y - 2x\frac{dy}{dx}\right)^2 = 4y\left(\frac{dy}{dx}\right)^3 \text{ order } = 1, \text{ degree } = 3$$

Question137

An ice ball melts at the rate which is proportional to the amount of ice at that instant. Half the quantity of ice melts in 20 minutes. x_0 is the initial quantity of ice. If after 40 minutes the amount of ice left is kx_0x, then k is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $\frac{1}{8}$

B. $\frac{1}{2}$

C. $\frac{1}{3}$

D. $\frac{1}{4}$

Answer: D

Solution:

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$$
$$\Rightarrow \frac{\mathrm{d}x}{x} = -kdt$$
$$\Rightarrow \log_e x = -kt + c$$
$$\Rightarrow x = e^{-kt+c} = e^c \cdot e^{-kt}$$
$$\text{at } t = 0, x^{-k} = x_0$$
$$\Rightarrow e^c = x_0$$
$$\Rightarrow x = x_0 e^{-kt}$$

Question138

The differential equation of family of lines, having x-intercept as a and y-intercept as b, is MHT CET 2022 (10 Aug Shift 1)

Options:

A. $\frac{d^2y}{dx^2} = 0$

B. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$

C. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = y$

D. $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Answer: A

Solution:

Diff $\frac{x}{a} + \frac{y}{b} = 1$ [two arbitrary constants]

$\frac{1}{a} + \frac{1}{b} \cdot \frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Again in diff $0 + \frac{1}{b} \cdot \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

$\Rightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0$

Question139

The solution of the differential equation $\mathrm{e}^{\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)} = x + 1; y(0) = 5, x \in (-1, \infty)$ MHT CET 2022 (08 Aug Shift 2)

Options:

A. $(x-1)\log(x+1)-x-5=y$

B. $(x+1)\log(x+1)+x+5=y$

C. $(x-1)\log(x+1)+x-5=y$

D. $(x+1)\log(x+1)-x+5=y$

Answer: D

Solution:

$$\begin{aligned}
&e^{\frac{dy}{dx}}=x+1\\
&\Rightarrow \frac{dy}{dx}=\log_e(x+1)\\
&\Rightarrow \int dy=\int \log_e(x+1)dx\\
&\Rightarrow y=(x+1)\log_e(x+1)-(x+1)+C \text{ [integrating by parts]}\\
&\text{putting } x=0 \text{ and } y=5 \text{ we get } C=6\\
&\Rightarrow y=(x+1)\log_e(x+1)-x+5
\end{aligned}$$

Question140

General solution of the differential equation $x\cos y\, dy=(xe^x\log x+e^x)\,dx$ is (where C is a constant of integration.) MHT CET 2022 (08 Aug Shift 2)

Options:

A. $\sin y=e^x\log x+C$

B. $\sin y=e^x+C\log x$

C. $\sin y=Ce^x+\log x$

D. $e^x\sin y=\log x+C$

Answer: A

Solution:

$$\begin{aligned}
&x\cos y\, dy=(xe^x\log x+e^x)\,dx\\
&\Rightarrow \int \cos y\, dy=\int e^x\left\{\log x+\frac{1}{x}\right\}dx\\
&\Rightarrow \sin y=e^x\log x+C\left[\because \int e^x\{f(x)+f'(x)\}\,dx=e^x f(x)+C\right]
\end{aligned}$$

Question141

The equation of the curve passing through the point $(0, -2)$ given that at any point (x, y) on the curve, the product of the slope of its tangent and y-co-ordinate of the point is equal to the x-co-ordinate of the point, is MHT CET 2022 (08 Aug Shift 2)

Options:

A. $y^2 + x^2 = 4$

B. $y^2 - x^2 = 4$

C. $2y^2 + x^2 = 8$

D. $4y^2 + 3x^2 = 16$

Answer: B

Solution:

$$A/Q \frac{dy}{dx} \cdot y = x$$
$$\Rightarrow \int y dy = \int x dx$$
$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + c$$

putting $x = 0$ and $y = -2$ we get $c = 2$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + 2$$
$$\Rightarrow y^2 - x^2 = 4$$

Question142

The general solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x + y - xy$ is (where C is a constant of integration) MHT CET 2022 (08 Aug Shift 1)

Options:

A. $\log(1 + y) = x + \frac{x^2}{2} + C$

B. $\log(1 - x) = \log(1 + y) + y + C$

C. $\log(1 + y) = y - \frac{x^2}{2} + C$

D. $\log(1 + y) = x - \frac{x^2}{2} + C$

Answer: D

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 - x + y - xy = (1-x)(1+y)$$
$$\Rightarrow \int \frac{\mathrm{d}y}{1+y} = \int (1-x)\mathrm{d}x$$
$$\Rightarrow \log(1+y) = x - \frac{x^2}{2} + C$$

Question143

The differential equation $y' = \frac{y}{x+\sqrt{xy}}$ has general solution given by (Where C is a constant of integration.) MHT CET 2022 (08 Aug Shift 1)

Options:

A. $y = Ce^{2\sqrt{xy}}$

B. $y = Ce^{2\left(\sqrt{\frac{y}{x}}\right)}$

C. $y = Ce^{2\left(\sqrt{\frac{x}{y}}\right)}$

D. $y = Ce^{-2\sqrt{xy}}$

Answer: C

Solution:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y}{x+\sqrt{xy}} \text{ let } y = vx$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = v + x \cdot \frac{\mathrm{d}v}{\mathrm{d}x}$$
$$\Rightarrow v + x \cdot \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{vx}{x+\sqrt{x \cdot vx}} = \frac{v}{1+\sqrt{v}}$$
$$\Rightarrow x \cdot \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{v}{1+\sqrt{v}} - v = \frac{-v\sqrt{v}}{1+\sqrt{v}}$$
$$\Rightarrow -\frac{1+\sqrt{v}}{v\sqrt{v}} \mathrm{d}v = \frac{\mathrm{d}x}{x}$$
$$\Rightarrow -\int \left(v^{\frac{-3}{2}} + v^{-1}\right) \mathrm{d}v = \int \frac{\mathrm{d}x}{x}$$
$$\Rightarrow 2v\frac{-1}{2} - \log v = \log x + \log c'$$
$$\Rightarrow \frac{2\sqrt{x}}{\sqrt{y}} - \log \frac{y}{x} = \log c'x$$
$$\Rightarrow \frac{2\sqrt{x}}{\sqrt{y}} = \log\left(c'x \cdot \frac{y}{x}\right)$$
$$\Rightarrow c \cdot e^{\frac{2\sqrt{x}}{\sqrt{y}}}$$

Question144

The particular solution of the differential equation $(2x-2y+3)\mathrm{d}x-(x-y+1)\mathrm{d}y=0$ when $x=0, y=1$ is MHT CET 2022 (08 Aug Shift 1)

Options:

A. $x-2y-\log(x-y+2)+2=0$

B. $x-y-\log(x-y+2)+1=0$

C. $2x+y-\log(x-y+2)-1=0$

D. $2x-y-\log(x-y+2)+1=0$

Answer: D

Solution:

Let $x-y+1=v$

$$\Rightarrow 1-\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}v}{\mathrm{d}x}$$
$$\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=1-\frac{\mathrm{d}v}{\mathrm{d}x}$$
$$\text{now } \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-2y+3}{x-y+1}$$
$$\Rightarrow 1-\frac{\mathrm{d}v}{\mathrm{d}x}=\frac{2v+1}{v}$$
$$\Rightarrow \frac{\mathrm{d}v}{\mathrm{d}x}=1-\frac{2v+1}{v}=\frac{-(v+1)}{v}$$
$$\Rightarrow \frac{v}{v+1}\mathrm{d}v=-\mathrm{d}x$$
$$\Rightarrow \int\left(1-\frac{1}{v+1}\right)\mathrm{d}v=-\int \mathrm{d}x$$
$$\Rightarrow n-\log|n+1|=-x+c$$
$$\Rightarrow (x-y+1)-\log(x-y+1+1)=-x+c$$
$$\Rightarrow 2x-y+1+\log(x-y+2)=c$$

$$\text{for } x=0, y=1; c=0$$
$$\Rightarrow 2x-y-\log(x-y+2)+1=0$$

Question145

The differential equation whose solution is $y=c^2+\frac{c}{x}$, where c is constant, is MHT CET 2022 (08 Aug Shift 1)

Options:

A. $x^4\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2-x\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$

B. $x^2\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+\frac{\mathrm{d}y}{\mathrm{d}x}-y=0$

C. $x\left(\frac{dy}{dx}\right)^2 - x^2\frac{dy}{dx} + y = 0$

D. $x^4\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} + y = 0$

Answer: A

Solution:

$$y = c^2 + \frac{c}{x}$$
$$\Rightarrow \frac{dy}{dx} = \frac{-c}{x^2}$$
$$\Rightarrow c = -x^2\frac{dy}{dx}$$

$$\text{Hence, } y = \left(-x^2\frac{dy}{dx}\right)^2 + \left(-x^2\frac{dy}{dx}\right) \times \frac{1}{x}$$
$$\Rightarrow y = x^4 \cdot \left(\frac{dy}{dx}\right)^2 - x \cdot \frac{dy}{dx}$$
$$\Rightarrow x^4\left(\frac{dy}{dx}\right)^2 - x \cdot \frac{dy}{dx} - y = 0$$

Question146

Water at $100^\circ\mathrm{C}$ cools in 10 minutes to $80^\circ\mathrm{C}$ in a room temperature of $25^\circ\mathrm{C}$, then the temperature of water after 20 minutes will be MHT CET 2022 (08 Aug Shift 1)

Options:

A. $65.33^\circ\mathrm{C}$

B. $69.33^\circ\mathrm{C}$

C. $60.33^\circ\mathrm{C}$

D. $63.33^\circ\mathrm{C}$

Answer: A

Solution:

$$\frac{dT}{dt} = -k(T-25)$$
$$\Rightarrow T - 25 = e^{-kt+c}$$
$$\Rightarrow T = 25 + e^{c} \cdot e^{-kt}$$
$$\text{for } t = 0, T = 100$$
$$\Rightarrow e^{c} = 75$$
$$\Rightarrow T = 25 + 75e^{-kt}$$
$$\text{for } t = 10, T = 80$$
$$\Rightarrow 80 = 25 + 75e^{-k\times 10}$$
$$\Rightarrow -10k = \log\left(\frac{11}{55}\right)$$
$$\text{i.e., } T = 25 + 75e^{\left(\frac{1}{10}\log\frac{11}{15}\right)t}$$
$$\text{Now for } t = 20, T = 25 + 75e^{20\times\frac{1}{10}\log\frac{11}{15}} = 25 + 75 \times \left(\frac{11}{15}\right)^{2}$$
$$\Rightarrow T = 25 + 40.33 = 65.33$$

Question147

The order and degree of the differential equation $\left(1+\frac{dy}{dx}\right)^{\frac{1}{3}} = \sqrt{\frac{d^2y}{dx^2}}$ are respectively. MHT CET 2022 (07 Aug Shift 2)

Options:

A. 3,1

B. 3,2

C. 2,3

D. 2,1

Answer: C

Solution:

$$\left(1+\frac{dy}{dx}\right)^{\frac{1}{3}} = \sqrt{\frac{d^2y}{dx^2}} \Rightarrow \left(1+\frac{dy}{dx}\right)^{2} = \left(\frac{d^2y}{dx^2}\right)^{3}$$

order $= 2$ and degree $= 3$.

Question148

The particular solution of $\frac{dy}{dx} = 1 + x + y^2 + xy^2$, when $y(0) = 0$. is MHT CET 2022 (07 Aug Shift 2)

Options:

A. $y = \log\left(1 + \frac{x^2}{2}\right)$

B. $y^3 = \log\left(1 + \frac{x^2}{2}\right)$

C. $y^2 = \tan\left(1 + \frac{x^2}{2}\right)$

D. $y = \tan\left(x + \frac{x^2}{2}\right)$

Answer: D

Solution:

$$\frac{dy}{dx} = 1 + x + y^2 + xy^2 = (1 + x)\left(1 + y^2\right)$$

$$\Rightarrow \int \frac{dy}{1 + y^2} = \int (1 + x) dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2} + c$$

$$\because y(0) = 0 \Rightarrow c = 0$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^2}{2}$$

$$\Rightarrow y = \tan\left(x + \frac{x^2}{2}\right)$$

Question149

The general solution of differential equation $xdy - ydx = 0$ represents MHT CET 2022 (07 Aug Shift 2)

Options:

A. the circle whose centre is at the origin

B. a straight line passing through the origin

C. a rectangular hyperbola

D. the parabola whose vertex is at the origin.

Answer: B

Solution:

$$xdy - ydx = 0 \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{x}$$
$$\Rightarrow \log y = \log x + \log c$$
$$\Rightarrow \log y = \log cx$$
$$\Rightarrow y = cx$$

which is a straight line passing through the origin

Question150

The differential equation $x^2(y+1)dx + y^2(x-1)dy = 0$ has the general solution given by (where C is a constant of integration.) MHT CET 2022 (07 Aug Shift 2)

Options:

A. $(x-1)^2 + (y-1)^2 + 2\log[(x+1)(y+1)] = C$

B. $(x-1)^2 + (y+1)^2 + 2\log[(x+1)(y-1)] = C$

C. $(x+1)^2 + (y+1)^2 + 2\log[(x-1)(y+1)] = C$

D. $(x+1)^2 + (y-1)^2 + 2\log[(x-1)(y+1)] = C$

Answer: D

Solution:

$$x^2(y+1)dx + y^2(x-1)dy = 0$$
$$\Rightarrow \int \frac{x^2}{1-x}dx = \int \frac{y^2}{y-1}dy$$
$$\Rightarrow \int\left(-x-1+\frac{1}{1+x}\right)dx = \int\left(y-1+\frac{1}{y+1}\right)dy$$
$$\Rightarrow -\frac{x^2}{2} - x - \log\left|1-x + C^1 = \frac{y^2}{2} - y + \log\right|y+1|$$
$$\Rightarrow C^1 = +x + -y + \log|1-x| + \log|y+1|$$
$$\Rightarrow 2C^1 = x^2 + 2x + y^2 - 2y + 2\log|(1-x)(y+1)|$$
$$\Rightarrow 2C^1 + 2 = (x+1)^2 + (y-1)^2 + 2\log|(1-x)(y+1)|$$
$$\Rightarrow (x+1)^2 + (y-1)^2 + 2\log|(x-1)(y+1)| = C$$

Question151

The particular solution of the differential equation $\frac{dy}{dx} - e^x = ye^x$, when $x = 0$ and $y = 1$ is MHT CET 2022 (07 Aug Shift 1)

Options:

A. $\log\left(\frac{y+1}{2}\right) = \frac{e^x}{2} - \frac{1}{2}$

B. $\log\left(\frac{y+1}{2}\right) = e^x - 1$

C. $\log(y-1) = e^x - 1$

D. $\log 2(y-1) = e^x - 1$

Answer: B

Solution:

$$\frac{dy}{dx} - e^x = ye^x \Rightarrow \frac{dy}{dx} = (y+1)e^x \Rightarrow \int \frac{dy}{y+1} = \int e^x dx$$
$$\Rightarrow \log(y+1) = e^x + c$$
$$\text{for } x = 0, y = 1 \Rightarrow c = \log 2 - 1$$
$$\text{Hence, } \log(y+1) = e^x + \log 2 - 1$$
$$\Rightarrow \log(y+1) - \log 2 = e^x - 1$$
$$\Rightarrow \log\left(\frac{y+1}{2}\right) = e^x - 1$$

Question152

The general solution of the differential equation $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$ is (where C is a constant of integration.) MHT CET 2022 (07 Aug Shift 1)

Options:

A. $2\left(x^2 - y^2\right) + x = C$

B. $x^2 + y^2 = Cx$

C. $x^2 - y^2 = Cx$

D. $x^2 + y^2 = Cy$

Answer: C

Solution:

$$x^2+y^2-2xy\frac{dy}{dx}=0 \Rightarrow \frac{dy}{dx}=\frac{x^2+y^2}{2xy} \Rightarrow v+x\cdot\frac{dv}{dx}=\frac{1+v^2}{2v}$$

[Let $y=nx$]

$$\Rightarrow \int \frac{2vdv}{1-v^2}=\int \frac{dx}{x}$$

$$\Rightarrow -\log\left|1-v^2\right|+\log|c|=\log|x|$$

$$\Rightarrow \log\left|\frac{c}{1-v^2}\right|=\log|x|$$

$$\Rightarrow \frac{c}{1-\frac{y^2}{x^2}}=x$$

$$\Rightarrow cx=x^2-y^2$$

Question153

The general solution of differential equation $e^{\frac{1}{2}\left(\frac{dy}{dx}\right)}=3^x$ is (where C is a constant of integration.) MHT CET 2022 (07 Aug Shift 1)

Options:

A. $y=x\log 3+C$

B. $y=x^2\log 3+C$

C. $y=2x\log 3+C$

D. $x=(\log 3)y^2+C$

Answer: B

Solution:

$$e^{\frac{1}{2}\left(\frac{dy}{dx}\right)}=3^x \Rightarrow \frac{1}{2}\frac{dy}{dx}=\log_e 3^x=x\log_e 3$$

$$\Rightarrow \frac{dy}{dx}=(2\log_e 3)x$$

$$\Rightarrow y=2\log_e 3\times\frac{x^2}{2}+c$$

$$\Rightarrow y=x^2\log_e 3+c$$

Question154

For the differential equation $\left[1-\left(\frac{dy}{dx}\right)^2\right]^{5/2}=8\frac{d^2y}{dx^2}$ has the order and degree MHT CET 2022 (07 Aug Shift 1)

Options:

A. 2 and 1

B. 2 and 3

C. 2 and 6

D. 2 and 2

Answer: B

Solution:

$$\left\{1-\left(\frac{dy}{dx}\right)^2\right\}^{\frac{5}{3}}=8\frac{dxy}{dx^2}$$

$$\Rightarrow\left\{1-\left(\frac{dy}{dx}\right)^2\right\}^5=8^3\left(\frac{dxy}{dx^2}\right)^3$$

$$\text{order }=2\text{ and degree }=3$$

Question155

The differential equation $\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$ determines a family of circles with MHT CET 2022 (06 Aug Shift 2)

Options:

A. variable radii and a fixed centre at $(0,-1)$

B. fixed radius of 1 unit and variable centres along the X-axis

C. fixed radius of 1 unit and variable centres along the Y-axis

D. variable radii and a fixed centre at $(0,1)$

Answer: B

Solution:

$$\frac{dy}{dx}=\frac{\sqrt{1-y^2}}{y}$$
$$\Rightarrow \int \frac{ydy}{\sqrt{1-y^2}}=\int dx$$
$$\Rightarrow -\sqrt{1-y^2}=x+c$$
$$\Rightarrow 1-y^2=(x+c)^2$$
$$\Rightarrow (x+c)^2+y^2=1$$

The above equation represents a circle having centre at $(-c, 0)$ which is variable and radius is equal to ' 1 ' which is fixed.

Question156

The general solution of the differential equation $\frac{dy}{dx}=\frac{3x+y}{x-y}$ is (where C is a constant of integration.) MHT CET 2022 (06 Aug Shift 2)

Options:

A. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right)-\log\left(\frac{y^2+3x^2}{x^2}\right)^{\frac{1}{2}}=\log(x)+C$

B. $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{y}{x\sqrt{3}}\right)+\log\left(\frac{y^2+3x^2}{x^2}\right)^{\frac{1}{2}}=\log(x)+C$

C. $\tan^{-1}\left(\frac{y}{x}\right)+\log\left(\frac{y^2+3x^2}{x^2}\right)=\log(x)+C$

D. $\tan^{-1}\left(\frac{x}{y}\right)+\log\left(\frac{y^2+3x^2}{x^2}\right)=\log(x)+C$

Answer: A

Solution:

$\frac{dy}{dx} = \frac{3x+y}{x-y}$

$$\Rightarrow v = x \cdot \frac{dv}{dx} = \frac{3+v}{1-v} \quad [\text{let } y = vx]$$
$$\Rightarrow x \cdot \frac{dv}{dx} = \frac{3+v}{1-v} - v$$
$$\Rightarrow x\frac{dv}{dx} = \frac{3+v^2}{1-v}$$
$$\Rightarrow \int \frac{1-v}{3+v^2} dv = \int \frac{dx}{x}$$
$$\Rightarrow \int \frac{dv}{3+v^2} - \frac{1}{2} \int \frac{2vdv}{3+v^2} = \int \frac{dx}{x}$$
$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{v}{\sqrt{3}} - \frac{1}{2} \log(3+v^2) = \log x + C$$

$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{x\sqrt{3}} - \frac{1}{2} \log\left(\frac{3x^2+y^2}{x^2}\right) = \log x + C$$
$$\Rightarrow \frac{1}{\sqrt{3}} \tan^{-1} \frac{y}{x\sqrt{3}} - \log\left(\frac{y^2+3x^2}{x^2}\right)^{\frac{1}{2}} = \log x + C$$

Question157

The solution of the differential equation $(1+e^{-x})(1+y^2)\frac{dy}{dx} = y^2$ which passes through the point $(0,1)$ is MHT CET 2022 (06 Aug Shift 1)

Options:

A. $y^2 + 1 = y\left(\log\left(\frac{1+e^x}{2}\right) + 2\right)$

B. $y^2 + 1 = y\left(\log\left(\left(\frac{1+e^{-x}}{2}\right) + 2\right)\right)$

C. $y^2 = 1 + y\log\left(\frac{1+e^{-x}}{2}\right)$

D. $y^2 = 1 + y\log\left(\frac{1+e^x}{2}\right)$

Answer: D

Solution:

$$(1+e^{-x})(1+y^2)\frac{dy}{dx}=y^2$$
$$\Rightarrow \int \frac{1+y^2}{y^2}dy=\int \frac{e^x}{1+e^x}dx$$
$$\Rightarrow -\frac{1}{y}+y=\log(1+e^x)+\log C$$
$$\Rightarrow -1+y^2=y\log c(1+e^x)$$

Putting $x=0$ and $y=1$ we get $c=\frac{1}{2}$

$$\Rightarrow -1+y^2=y\log\left(\frac{1+e^x}{2}\right)$$
$$\Rightarrow y^2=1+y\log\left(\frac{1+e^x}{2}\right)$$

Question158

The assets of a person are increasing at a rate proportional to the square root of the assets at a given time. His assets increase from Rupees 9 crores to Rupees 16 crores in 2 years, then his assets at the end of 5 more years will be MHT CET 2022 (06 Aug Shift 1)

Options:

A. 46.25 cores

B. 42.25 crores

C. 30.25 crores

D. 56.25 crores

Answer: B

Solution:

$$\frac{dx}{dt}\propto\sqrt{x}$$
$$\Rightarrow \frac{dx}{dt}=k\sqrt{x}$$
$$\Rightarrow \frac{dx}{\sqrt{x}}=kdt$$
$$\Rightarrow 2\sqrt{x}=kt+C$$

When $t=0, x=9$ i.e. $2\sqrt{9}=k\times0+C$

$$\Rightarrow c=6$$
$$\Rightarrow 2\sqrt{x}=kt+6[\text{ Putting } c=6]$$
When $t=2, x=16$ i.e. $2\sqrt{16}=k\times2+6\Rightarrow k=1$
$$\Rightarrow 2\sqrt{x}=t+6\ [\text{Putting } k=1]$$

After 5 years at $t = 7$

$$2\sqrt{x} = 7 + 6$$
$$\Rightarrow \sqrt{x} = 6.5$$
$$\Rightarrow x = 42.25$$

Question159

The particular solution of $\log\left(\frac{dy}{dx}\right) = 3x + 4y$ at $x = y = 0$ is MHT CET 2022 (06 Aug Shift 1)

Options:

A. $3e^{-4y} - 4e^{3x} = 7$

B. $3e^{-4y} + 4e^{3x} = 7$

C. $4e^{-4y} - 3e^{3x} = 7$

D. $4e^{-4y} + 3e^{3x} = 7$

Answer: B

Solution:

$$\log\left(\frac{dy}{dx}\right) = 3x + 4y \text{ and } x = y = 0$$
$$\Rightarrow \frac{dy}{dx} = e^{3x+4y}$$
$$\Rightarrow \int e^{-4y} dy = \int e^{3x} dx$$
$$\Rightarrow \frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} + C$$

Putting $x = 0$ and $y = 0$ we get $C = \frac{-7}{12}$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$
$$\Rightarrow 3e^{-4y} = -4e^{3x} + 7$$
$$\Rightarrow 3e^{-4y} + 4e^{3x} = 7$$

Question160

General solution of the differential equation $\left(y^3 + y\right)\left(x^2 + 1\right) dy = \left(xy^4 + 2y^2 x\right) dx$ is (where C is a constant of integration.) MHT CET 2022 (05 Aug Shift 2)

Options:

A. $y^2\left(y^2 + 1\right) = C\left(x^2 + 1\right)^2$

B. $y^2\left(y^2 + 2\right) = C\left(x^2 + 1\right)$

C. $y^2\left(y^2+2\right)=C\left(x^2+1\right)^2$

D. $y^2\left(y^2+1\right)=C\left(x^2+2\right)^2$

Answer: C

Solution:

$$\begin{aligned}
&\left(y^3+y\right)\left(x^2+1\right) dy=\left(x y^4+2 y^2 x\right) dx \\
&\Rightarrow \int \frac{y^3+y}{y^4+2 y^2} dy=\int \frac{x}{x^2+1} dx \\
&\Rightarrow \frac{1}{4} \log_e\left(y^4+2 y^2\right)=\frac{1}{2} \log_e\left(x^2+1\right)+\log C^1 \\
&\Rightarrow \log_e\left(y^4+2 y^2\right)=2 \log_e\left(C^1\right)^2\left(x^2+1\right) \\
&\Rightarrow y^4+2 y^2=\left(c^1\right)^4\left(x^2+1\right)^2 \\
&\Rightarrow y^2\left(y^2+2\right)=C\left(x^2+1\right)^2
\end{aligned}$$

Question161

The particular solution of the differential equation $\frac{dy}{dx}=e^{2y}\cos x$, when $y\left(\frac{\pi}{6}\right)=0$ is
MHT CET 2022 (05 Aug Shift 2)

Options:

A. $\sin x-\frac{e^{2y}}{2}=0$

B. $4\sin x-e^{-2y}-1=0$

C. $\sin x+e^{-2y}-2=0$

D. $2\sin x+e^{-2y}-2=0$

Answer: D

Solution:

$$\begin{aligned}
&\frac{dy}{dx}=e^{2y}\cos x \Rightarrow \int e^{2y} dy=\int \cos dx \\
&\Rightarrow \frac{e^{2y}}{-2}=\sin x+c
\end{aligned}$$

Putting $x=\frac{\pi}{6}$ and $y=0$ we get $c=-1$

$$\begin{aligned}
&\Rightarrow \frac{e^{-2y}}{-2}=\sin x-1 \\
&\Rightarrow e^{-2y}=-2\sin x+2 \\
&\Rightarrow 2\sin x+e^{-2y}-2=0
\end{aligned}$$

Question162

The general solution of differential equation $\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y}$ is (where C is a constant of integration.) MHT CET 2022 (05 Aug Shift 2)

Options:

A. $\mathrm{e^x + e^{-y} + \frac{1}{3} e^{x^3} = C}$

B. $e^x - e^{-y} - \frac{1}{3} e^{x^3} = C$

C. $e^x - e^{-y} + \frac{1}{3} e^{x^3} = C$

D. $e^x - e^{-y} + \frac{1}{3} e^{x^3} = C$

Answer: A

Solution:

$$\begin{aligned} &\frac{dy}{dx} = e^{x+y} + x^2 e^{x^3+y} \\ &\Rightarrow \frac{dy}{dx} = e^y\left(e^x + x^2 e^{x^3}\right) \\ &\Rightarrow \int e^{-y} dy = \int\left(e^x + x^2 e^{x^3}\right) dx \\ &\Rightarrow -e^{-y} + c = e^x + \frac{1}{3} e^{x^3} \\ &\Rightarrow e^x + e^{-y} + \frac{1}{3} e^{x^3} = c \end{aligned}$$

Question163

If $y = y(x)$ and $\frac{2+\sin x}{y+1}\left(\frac{dy}{dx}\right) = -\cos x, y(0) = 1$, then $y\left(\frac{\pi}{2}\right)$ is equal to MHT CET 2022 (05 Aug Shift 1)

Options:

A. $\frac{2}{3}$

B. $\frac{1}{3}$

C. $-\frac{1}{3}$

D. 1

Answer: B

Solution:

$$\frac{2+\sin x}{y+1}\frac{dy}{dx}=-\cos x$$
$$\Rightarrow \int \frac{dy}{y+1}=\int \frac{-\cos x dx}{2+\sin x}$$
$$\Rightarrow \log_e|y+1|=-\log_e|2+\sin x|+\log_e C$$
$$\Rightarrow \log_e|y+1|+\log_e|2+\sin x|=\log_e C$$
$$\Rightarrow \log_e|(y+1)(2+\sin x)|=\log_e C$$
$$\Rightarrow (y+1)(2+\sin x)=C$$
Putting $x=0$ and $y=1$ we get $c=4$
$$\Rightarrow (y+1)(2+\sin x)=4$$

Now putting $x=\frac{\pi}{2}$ we get $y=\frac{1}{3}$

Question164

General solution of the differential equation $\sin^3 x\frac{dx}{dy}=\sin y$ is given by MHT CET 2022 (05 Aug Shift 1)

Options:

A. $\cos y-\frac{3}{4}\cos x-\frac{1}{12}\cos 3x=C$

B. $\cos y-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x=C$

C. $\cos y+\frac{3}{4}\cos x-\frac{1}{12}\cos 3x=C$

D. $\cos y+\frac{3}{4}\cos x-\frac{1}{12}\cos 3x=C$

Answer: B

Solution:

$$\sin^3 x\frac{dx}{dy}=\sin y\Rightarrow\int\sin^3 x dx=\int\sin y dy$$
$$\Rightarrow\int\frac{3\sin x-\sin 3x}{4}dx=\int\sin y dy$$
$$\Rightarrow\frac{-3}{4}\cos x+\frac{1}{4}\cdot\frac{\cos 3x}{3}=-\cos y+C$$
$$\Rightarrow\cos y-\frac{3}{4}\cos x+\frac{1}{12}\cos 3x=C$$

Question165

A curve passes through the point $\left(1, \frac{\pi}{6}\right)$. Let the slope of the curve at each point (x, y) be given by $\frac{y}{x} + \sec\left(\frac{y}{x}\right), x > 0$, then the equation of the curve is MHT CET 2022 (05 Aug Shift 1)

Options:

A. $\sin\left(\frac{y}{x}\right) = \log x + \frac{1}{2}$

B. $\operatorname{cosec}\left(\frac{y}{x}\right) = \log e + 2$

C. $\cos\left(\frac{2y}{x}\right) = \log x + \frac{1}{2}$

D. $\sec\left(\frac{2y}{x}\right) = \log x + 2$

Answer: A

Solution:

Here $\frac{dy}{dx} = \frac{y}{x} + \sec\left(\frac{y}{x}\right)$

Let $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x \cdot \frac{dv}{dx}$

$$\Rightarrow v + x \cdot \frac{dv}{dx} = v + \sec v$$
$$\Rightarrow x \cdot \frac{dv}{dx} = \sec v$$
$$\Rightarrow \int \cos v \cdot dv = \int \frac{dx}{x}$$
$$\Rightarrow \sin v = \log_e x + C$$

$\Rightarrow \sin\left(\frac{y}{x}\right) = \log_e x + C$

Putting $x = 1, y = \frac{\pi}{6} \Rightarrow c = \frac{1}{2}$

$\Rightarrow \sin\left(\frac{y}{x}\right) = \log_e^x + \frac{1}{2}$

Question166

A population P grew at the rate given by the equation $\frac{dp}{dt} = 0.05P$, then the population will become double in MHT CET 2021 (24 Sep Shift 2)

Options:

A. $20(\log 2)$ years

B. $10(\log 2)$ years

C. $5(\log 2)$ years

D. $12(\log 2)$ years

Answer: A

Solution:

$$\frac{dp}{dt} = 0.05P$$
$$\therefore \quad \int \frac{dp}{0.05p} = \int dt \Rightarrow 20 \int \frac{dp}{P} = \int dt$$
$$\therefore \quad 20 \log |P| = t + c$$
When $t = 0, P = P \Rightarrow c = 20 \log |P|$
$$\therefore 20 \log |P| = t + 20 \log |P|$$

When $\mathrm{t} = 0, \mathrm{P} = \mathrm{P} \Rightarrow \mathrm{c} = 20 \log |\mathrm{P}|$

$$\therefore 20 \log |\mathrm{P}| = \mathrm{t} + 20 \log |\mathrm{P}|$$

When population doubles, we write

$$20 \log |2\mathrm{P}| = \mathrm{t} + 20 \log |\mathrm{P}|$$
$$\therefore \quad \mathrm{t} = 20 \log |2\mathrm{P}| - 20 \log |\mathrm{P}| = 20[\log |2\mathrm{P}| - \log |\mathrm{P}|]$$
$$= 20 \left[\log \left|\frac{2\mathrm{P}}{\mathrm{P}}\right|\right]$$
$$= 20(\log 2\ ) \text{ years}$$

Question167

The differential equation of all parabolas whose axis is y-axis, is MHT CET 2021 (24 Sep Shift 2)

Options:

A. $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

B. $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

C. $x\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

D. $\frac{d^2y}{dx^2} - y = 0$

Answer: C

Solution:

Differential Equation of parabolas whose axis is Y axis, is Let the vertex be $(0, \mathrm{k})$

$$(x-0)^2 = 4b(y-k) \quad \ldots (1)$$

$$\therefore x^2 = 4by - 4bk$$

$$\therefore 2x = 4b\frac{dy}{dx} - 0 \Rightarrow b = \left(\frac{x}{2}\right) \times \frac{1}{\left(\frac{dy}{dx}\right)}$$

$$2x = 4b\frac{dy}{dx} - 0 \Rightarrow b = \left(\frac{x}{2}\right) \times \frac{1}{\left(\frac{dy}{dx}\right)}$$

Substituting value of b in eq. (1), we get

$$\therefore \quad x^2 = 4\left(\frac{x}{2}\right) \times \frac{1}{\left(\frac{dy}{dx}\right)}(y-k)$$

$$\therefore \quad \mathrm{x}^2\left(\frac{\mathrm{dy}}{\mathrm{dx}}\right) = 2\mathrm{x}(\mathrm{y}-\mathrm{k})$$

$$\therefore \quad x^2\left(\frac{d^2y}{dx^2}\right) + \left(\frac{dy}{dx}\right)(2x) = 2x\frac{dy}{dx} + 2(y-k)$$

$$\therefore \quad \mathrm{y}-\mathrm{k} = \frac{\mathrm{x}^2\left(\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dx}}\right)}{2}$$

Substituting value of $(\mathrm{y}-\mathrm{k})$ in eq. (1), we get

$$x^2 = 4\left(\frac{x}{2}\right)\left[\frac{1}{\left(\frac{dy}{dx}\right)}\right]^{\frac{x^2\left(\frac{d^2y}{d^2}\right)}{2}]}]$$

$$\therefore \quad x^2\left(\frac{dy}{dx}\right) = x^3\left(\frac{d^2y}{dx^2}\right) \Rightarrow x\left(\frac{d^2y}{dx^2}\right) - \left(\frac{dy}{dx}\right) = 0$$

Question168

The general solution of the differential equation $\frac{\mathrm{dy}}{\mathrm{dx}} = \tan\left(\frac{\mathrm{y}}{\mathrm{x}}\right) + \frac{\mathrm{y}}{\mathrm{x}}$ is MHT CET 2021 (24 Sep Shift 2)

Options:

A. $\sin\left(\frac{y}{x}\right) = cy$

B. $\cos\left(\frac{y}{x}\right) = cy$

C. $\cos\left(\frac{y}{x}\right) = cx$

D. $\sin\left(\frac{y}{x}\right) = cx$

Answer: D

Solution:

$$\frac{dy}{dx}=\tan\left(\frac{y}{x}\right)+\frac{y}{x}$$

$$\text{Put } \frac{y}{x}=v \Rightarrow y=xv \Rightarrow \frac{dy}{dx}=x\frac{dv}{dx}+v$$

$$\therefore \quad v+x\frac{dv}{dx}=\tan v+v \Rightarrow x\frac{dv}{dx}=\tan v$$

$$\therefore \quad \int\frac{dv}{\tan v}=\int\frac{dx}{x}$$

$$\therefore \quad \log|\sin v|=\log|x|+\log|c| \Rightarrow \sin v=xc$$

$$\therefore \quad \sin\left(\frac{y}{x}\right)=xc$$

Question169

The particular solution of the differential equation $\left(1+e^{2x}\right)dy+e^{x}\left(1+y^{2}\right)dx=0$ at $x=0$ and $y=1$ is MHT CET 2021 (24 Sep Shift 2)

Options:

A. $\tan^{-1}\mathrm{e}^{\mathrm{x}}-\tan^{-1}\mathrm{y}=0$

B. $\tan^{-1}\mathrm{e}^{\mathrm{x}}+\tan^{-1}\mathrm{y}=\frac{\pi}{2}$

C. $\tan^{-1}\mathrm{e}^{\mathrm{x}}+\tan^{-1}\mathrm{y}=\frac{3\pi}{4}$

D. $\tan^{-1}e^{x}-\tan^{-1}y=\frac{3\pi}{4}$

Answer: B

Solution:

$$\left(1+e^{2x}\right) dy + e^{x}\left(1+y^{2}\right) dx = 0$$
$$\therefore \frac{dy}{1+y^{2}} + \frac{e^{x}}{\left(1+e^{2x}\right)} dx = 0$$
$$\therefore \int \frac{dy}{1+y^{2}} = -\int \frac{e^{x}}{1+e^{2x}} dx$$

Put $e^{x} = t \Rightarrow e^{x} dx = dt$

$\therefore \int \frac{dy}{1+y^{2}} = -\int \frac{dt}{1+t^{2}} \Rightarrow \tan^{-1}(y) = -\tan^{-1}(t) + c$

$\therefore \tan^{-1}(y) + \tan^{-1}(e^{x}) = c$

We have $x = 0, y = 1$

$\therefore \tan^{-1}(1) + \tan^{-1}(e^{\circ}) = c \Rightarrow c = 2\tan^{-1}(1) = \frac{\pi}{2}$

$\therefore \tan^{-1} y + \tan^{-1} e^{x} = \frac{\pi}{2}$

Question170

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$ are respectively. MHT CET 2021 (24 Sep Shift 2)

Options:

A. 2 and 2

B. 1 and 2

C. 1 and 1

D. 2 and 1

Answer: B

Solution:

$$\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$$
$$\therefore \sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x \text{ and squaring both sides, we get}$$
$$\frac{dy}{dx} = 16\left(\frac{dy}{dx}\right)^{2} + 56x\left(\frac{dy}{dx}\right) + 49x^{2}$$
$$\therefore \text{ Order } = 1, \text{ degree } = 2$$

Question171

The general solution of the differential equation $\cos(x+y)\frac{dy}{dx}=1$ is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $y=\tan(x+y)+c$

B. $y=\sec(x+y)+c$

C. $y=\tan\left(\frac{x+y}{2}\right)+c$

D. $y=\cot\left(\frac{x+y}{2}\right)+c$

Answer: C

Solution:

$$\cos(x+y)\frac{dy}{dx}=1$$

Put $x+y=V \Rightarrow 1+\frac{dy}{dx}=\frac{dV}{dx}$

$$\therefore \cos \mathrm{V}\left(\frac{\mathrm{dV}}{\mathrm{dx}}-1\right)=1 \Rightarrow \cos \mathrm{V}\left(\frac{\mathrm{dV}}{\mathrm{dx}}\right)=1+\cos \mathrm{V}$$

$$\therefore \int \frac{\cos \mathrm{V}}{1+\cos \mathrm{V}} \mathrm{dV}=\int \mathrm{dx}$$

$$\therefore \int\left[\frac{1+\cos \mathrm{V}}{1+\cos \mathrm{V}}-\frac{1}{1+\cos \mathrm{V}}\right] \mathrm{dV}$$

$$=\int \mathrm{dx} \Rightarrow \int \mathrm{dV}-\frac{1}{2} \int \sec^2 \frac{\mathrm{V}}{2} \mathrm{dV}=\int \mathrm{dx}$$

$$\therefore \mathrm{V}-\frac{1}{2} \frac{\tan\left(\frac{\mathrm{V}}{2}\right)}{\left(\frac{1}{2}\right)}=\mathrm{x}+\mathrm{c} \Rightarrow \mathrm{V}-\tan\left(\frac{\mathrm{V}}{2}\right)=\mathrm{x}+\mathrm{c}$$

$$\Rightarrow \mathrm{x}+\mathrm{y}-\tan\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)=\mathrm{x}+\mathrm{c}$$

$$\mathrm{y}=\tan\left(\frac{\mathrm{x}+\mathrm{y}}{2}\right)+\mathrm{c}$$

Question172

Radium decompose at the rate proportional to the amount present at any time. If $P\%$ of amount disappears in one year, then amount of radium left after 2 years is MHT CET

2021 (24 Sep Shift 1)

Options:

A. $\left(10-\frac{P}{10}\right)^2$

B. $x_0\left[1+\frac{P}{100}\right]^2$

C. $x_0\left[1-\frac{P}{100}\right]^2$

D. $x_0\left[10-\frac{P}{100}\right]^2$

Answer: C

Solution:

$\mathrm{P}\%$ amount disappears in one year.

Let initial amount of radium $=\mathrm{x}_0$

$\therefore$ Amount left after 1 year $=\mathrm{x}_0-\frac{\mathrm{P}}{100}\times\mathrm{x}_0=\mathrm{x}_0\left(1-\frac{\mathrm{P}}{100}\right)$

Amount left after 2 years

$$=x_0\left(1-\frac{P}{100}\right)-\frac{P}{100}\times x_0\left(1-\frac{P}{100}\right)$$
$$=x_0\left(1-\frac{P}{100}\right)\left(1-\frac{P}{100}\right)=x_0\left(1-\frac{P}{100}\right)^2$$

Question173

The differential equation obtained by eliminating A and B from $y=A\cos\omega t+B\sin\omega t$ MHT CET 2021 (24 Sep Shift 1)

Options:

A. $\frac{d^2y}{dt^2}+\omega^2y=0$

B. $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{dt}^2}+\omega\mathrm{y}^2=0$

C. $\frac{d^2y}{dt^2}-\omega^2y=0$

D. $\frac{d^2y}{dt^2}-\omega y^2=0$

Answer: A

Solution:

$$y = A\cos\omega t + B\sin\omega t$$

$$\therefore \quad \frac{dy}{dt} = -A\omega\sin\omega t + B\omega\cos\omega t$$

$$\frac{d^2y}{dt^2} = -A\omega^2\cos\omega t - B\omega^2\sin\omega t$$

$$= -\omega^2(A\cos\omega t + B\sin\omega t) = -\omega^2 y$$

$$\therefore \quad \frac{d^2y}{dt^2} + \omega^2 y = 0$$

Question174

The particular solution of the differential equation $y(1+\log x)\frac{dx}{dy} - x\log x = 0$ when $x = e, y = e^2$ is MHT CET 2021 (24 Sep Shift 1)

Options:

A. $y^2 = e^4\log x$

B. $y = e^2\log x$

C. $y = x^2\log x$

D. $y = ex\log x$

Answer: D

Solution:

$$y(1+\log x)\frac{dy}{dx} - x\log x = 0$$

$$\therefore y(1+\log x)\frac{dy}{dx} = x\log x \Rightarrow \frac{dy}{dx} = \frac{y(1+\log x)}{x\log x}$$

$$\therefore \int \frac{dy}{dx} = \int \frac{1+\log x}{x\log x}dx$$

$$\therefore \log|y| + \log|x\log x| + \log c$$

$$\text{We have } x = e, y = e^2$$

$$\therefore \log|e|^2 = \log|e\log e| + \log c$$

$$2 = 1 + \log c \Rightarrow \log c = 1 = \log e$$

$$\therefore \log|y| = \log|x\log x| + \log e$$

$$y = ex\log x$$

Question175

The order and degree of the differential equation $\frac{d^2y}{dx^2} = \sqrt{\frac{dy}{dx}}$ are respectively MHT CET 2021 (24 Sep Shift 1)

Options:

A. 2,3

B. 3,3

C. 2,2

D. 1,3

Answer: C

Solution:

$$\frac{d^2y}{dx^2} = \sqrt{\frac{dy}{dx}}$$

$$\therefore \quad \left(\frac{d^2y}{dx^2}\right) = \frac{dy}{dx}$$

$$\therefore \text{ order } = 2 \text{ and degree } = 2$$

Question176

The general solution of the differential equation $\frac{dy}{dx} = 2^{y-x}$ is MHT CET 2021 (23 Sep Shift 2)

Options:

A. $2^x - 2^y = c$

B. $\frac{1}{2^x} - \frac{1}{2^y} = c$

C. $\frac{1}{2^x} + \frac{1}{2^y} = c$

D. $2^x + 2^y = c$

Answer: B

Solution:

$$\frac{dy}{dx}=2^{y-x}$$

$$\therefore \frac{dy}{dx}=\frac{2^y}{2^x} \Rightarrow \int \frac{dy}{2^y}=\int \frac{dx}{2^x}$$

$$\therefore \int 2^{-y} dy=\int 2^{-x} dx \Rightarrow \frac{2^{-y}}{-\ell n 2}=\frac{2^{-x}}{-\ln 2}+c_1 \Rightarrow \frac{2^{-y}}{\ln 2}=\frac{2^{-x}}{\ln 2}-c_1$$

$$\therefore \frac{1}{\ln 2}\left[2^{-x}-2^{-y}\right]=c_1$$

$$\therefore \frac{1}{2^x}-\frac{1}{2^y}=c, \text{ where } c=\left(c_1\right)(\ln 2)$$

Question177

The general solution of the differential equation $\frac{dx}{dt}=\frac{x \log x}{t}$ is MHT CET 2021 (23 Sep Shift 2)

Options:

A. lot $x-x=c$

B. $e^{ct}+x=0$

C. $\log t=x+c$

D. $e^{ct}=x$

Answer: D

Solution:

$$\frac{dx}{dt}=\frac{x \log x}{t}$$

$$\therefore \int \frac{dx}{x \log x}=\int \frac{dt}{t}$$

$$\therefore \log |\log x|=\log |t|+\log c$$

$$\therefore \log x=tc \Rightarrow x=e^{tc}$$

Question178

The general solution of the differential equation $\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$ is MHT CET 2021 (23 Sep Shift 2)

Options:

A. $x+y+1=c(1+x+y+2xy)$

B. $x+y+1=c(2+x+y+2xy)$

C. $x+y+1=c(1-x-y-2xy$

D. $x+y+2=c(2-x-y-2xy)$

Answer: C

Solution:

$$\frac{dy}{dx}+\frac{y^2+y+1}{x^2+x+1}=0$$

$$\therefore \frac{dy}{dx}=-\left(\frac{y^2+y+1}{x^2+x-1}\right)$$

$$\therefore \int\frac{dy}{y^2+y+1}=-\int\frac{dx}{x^2+x+1}$$

$$\Rightarrow \int\frac{dy}{y^2+y+\frac{1}{4}+\frac{3}{4}}=-\int\frac{dx}{x^2+x+\frac{1}{4}+\frac{3}{4}}$$

$$\int\frac{dy}{\left(y+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}=-\int\frac{dx}{\left(x+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$\Rightarrow \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2}\tan^{-1}\left[\frac{y+\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)}\right]=\frac{-1}{\left(\frac{\sqrt{3}}{2}\right)^{-1}\tan^{-1}}\left[\frac{x+\frac{1}{2}}{\left(\frac{\sqrt{3}}{2}\right)}\right]+C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2\text{y}+1}{\sqrt{3}}\right)=\frac{-2}{\sqrt{3}}\tan^{-1}\left(\frac{2\text{x}+1}{\sqrt{3}}\right)+\text{C}_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\frac{(2\text{y}+1)}{\sqrt{3}}+\left(\frac{2\text{x}+1}{\sqrt{3}}\right)}{1-\left(\frac{2\text{y}+1}{\sqrt{3}}\right)\left(\frac{2\text{x}+1}{\sqrt{3}}\right)}\right]=\text{C}_1$$

$$\Rightarrow \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\frac{2(x+y+1)}{\sqrt{3}}}{\frac{3-(4xy+2x+2y+1)}{3}}\right]=C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{2(x+y+1)}{\sqrt{3}}\times\frac{3}{2(1-2xy-x-y)}\right]=C_1$$

$$\therefore \frac{2}{\sqrt{3}}\tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{1-2xy-x-y}\right]=C_1$$

$$\Rightarrow \tan^{-1}\left[\frac{\sqrt{3}(x+y+1)}{1-x-y-2xy}\right]=C_2$$

$$\ldots\left[\text{where } \mathrm{C}_2=\frac{\sqrt{3}\mathrm{C}_1}{2}\right]$$

$$\therefore \frac{\sqrt{3}(x+y+1)}{1-x-y-2xy}=\tan C_2 \Rightarrow \frac{x+y+1}{1-x-y-2xy}=\frac{\tan C_2}{\sqrt{3}}=c$$

... (say)

$$\therefore \mathrm{x}+\mathrm{y}+1=\mathrm{c}(1-\mathrm{x}-\mathrm{y}-2\mathrm{xy})$$

Question179

The particular solution of differential equation $(\mathrm{x}+\mathrm{y})\mathrm{dy}+(\mathrm{x}-\mathrm{y})\mathrm{dx}=0$ **at** $\mathrm{x}=\mathrm{y}=1$ **is MHT CET 2021 (23 Sep Shift 2)**

Options:

A. $\log\left|\frac{\mathrm{x}^2+\mathrm{y}^2}{2}\right|=\frac{\pi}{2}-2\tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

B. $\log\left|\mathrm{x}^2+\mathrm{y}^2\right|=\frac{\pi}{2}-2\tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

C. $\log\left|\frac{\mathrm{x}^2+\mathrm{y}^2}{2}\right|=\frac{\pi}{4}-\tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

D. $\log\left|\mathrm{x}^2+\mathrm{y}^2\right|=\frac{\pi}{4}-2\tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right)$

Answer: A

Solution:

$$(x+y)dy+(x-y)dx=0$$

We have $\frac{dy}{dx}=-\frac{(x-y)}{x+y}$ Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore v+x\frac{dv}{dx}=\frac{-(x-vx)}{x+vx}=\frac{-x(1-v)}{x(1+v)}=\frac{-(1-v)}{1+v}$$

$\therefore x\frac{dv}{dx}=\frac{-1+v}{1+v}-v=\frac{-1+v-v-v^2}{1+v} \Rightarrow x\frac{dv}{dx}=\frac{-\left(1+v^2\right)}{1+v}$

$$\therefore \int \frac{(1+v)}{1+v^2} dv = -\int \frac{dx}{x} \Rightarrow \int \frac{dv}{1+v^2} + \frac{1}{2} \int \frac{2v}{1+v^2} dv = -\int \frac{dx}{x}$$

$$\therefore \tan^{-1}(\mathrm{v}) + \frac{1}{2} \log\left|1+\mathrm{v}^2\right| = -\log \mathrm{x} + \log \mathrm{c}_1$$

$$\Rightarrow \tan^{-1}\left(\frac{\mathrm{y}}{\mathrm{x}}\right) + \frac{1}{2} \log\left|1+\frac{\mathrm{y}^2}{\mathrm{x}^2}\right| = -\log \mathrm{x} + \log \mathrm{c}_1$$

$$2 \tan^{-1}\left(\frac{y}{x}\right) + \log\left|\frac{x^2+y^2}{x^2}\right| + 2 \log x = \log c \ldots [\because \log c = 2 \log c_1]$$

$$\therefore 2 \tan^{-1}\left(\frac{y}{x}\right) + \log\left|\frac{x^2+y^2}{x^2} \times x^2\right| = \log c$$

Question180

If the surrounding air is kept at $25^{\circ}\mathrm{C}$ and a body cools from $80^{\circ}\mathrm{C}$ to $50^{\circ}\mathrm{C}$ in 30 minutes, then temperature of the body after one hour will be MHT CET 2021 (23 Sep Shift 2)

Options:

A. $31.72^{\circ}\mathrm{C}$ approximately

B. $34.74^{\circ}\mathrm{C}$ approximately

C. $32.36^{\circ}\mathrm{C}$ approximately

D. 36.36° approximately

Answer: D

Solution:

By Newton's law of cooling, we write

$$\frac{d\theta}{dt} \propto (\theta - \theta_0)$$
$$\therefore \frac{d\theta}{dt} = k(\theta - \theta_0) \Rightarrow \int\left(\frac{d\theta}{\theta - \theta_0}\right) = \int kdt$$
$$\therefore \log|\theta - \theta_0| = kt + c$$
When $t = 0, \theta = 80$ and $\theta_0 = 25$
$$\therefore \log|80 - 25| = 0 + c \quad \Rightarrow c = \log|55|$$
When $t = 30, \theta = 50$
$$\therefore \log|50 - 25| = 30k + \log|55|$$
$$\therefore k = \frac{1}{30}\log\left|\frac{5}{11}\right|$$

From (1), (2), (3) we write

$$\log|\theta - \theta_0| = \frac{1}{30}\log\left|\frac{5}{11}\right|t + \log|55|$$

When $t = 60$, we get

$$\log|\theta - \theta_0| = \frac{60}{30}\log\left|\frac{5}{11}\right| + \log|55|$$
$$= \log\left(\left|\frac{5}{11}\right|\right)^2 + \log|55| = \log\left|\frac{25}{121} \times 55\right| = \log\left|\frac{125}{11}\right|$$
$$\therefore \theta - 25 = \frac{125}{11} \Rightarrow \theta = 36.36^\circ C$$

Question181

The velocity of a particle at time t is given by the relation $v = 6t - \frac{t^2}{6}$. Its displacement S is zero at $t = 0$, then the distance travelled in 3sec is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $\frac{51}{2}$ units

B. $\frac{39}{2}$ units

C. $\frac{57}{2}$ units

D. $\frac{33}{2}$ units

Answer: A

Solution:

$\mathrm{v} = 6\mathrm{t} - \frac{\mathrm{t}^2}{6}$ and we know that $\mathrm{v} = \frac{\mathrm{ds}}{\mathrm{dt}}$

$$\therefore \int \mathrm{d}s = \int \left(6\mathrm{t} - \frac{\mathrm{t}^2}{6}\right) \mathrm{dt}$$

$$\therefore \mathrm{s} = \frac{6\mathrm{t}^2}{2} - \frac{\mathrm{t}^3}{6(3)} + \mathrm{c} \Rightarrow \mathrm{s} = 3\mathrm{t}^2 - \frac{\mathrm{t}^3}{18} + \mathrm{c}$$

We know that $\mathrm{s} = 0$, when $\mathrm{t} = 0 \Rightarrow \mathrm{c} = 0$

$$\therefore \mathrm{s} = 3\mathrm{t}^2 - \frac{\mathrm{t}^3}{18} \Rightarrow (\mathrm{s})_{\mathrm{t}=3} = 3(3)^2 - \frac{(3)^3}{18} = \frac{51}{2} \text{ units}$$

Question182

The particular solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ when $\mathrm{x} = \frac{2}{3}$ and $\mathrm{y} = \frac{1}{3}$ is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $2x + 2y - 2 = \log|x + y|$

B. $\mathrm{y} - \mathrm{x} + \frac{1}{3} = \log|\mathrm{x} + \mathrm{y}|$

C. $x + y - 1 = \log|x + y|$

D. $4x - 5y - 1 = \log|x + y|$

Answer: B

Solution:

$$\frac{dy}{dx} = \frac{x + y + 1}{x + y - 1}$$

Put $x + y = v \Rightarrow 1 + \frac{dy}{dx} = \frac{dv}{dx}$

$$\therefore \frac{dv}{dx}-1=\frac{v+1}{v-1} \Rightarrow \frac{dv}{dx}=\frac{v+1}{v-1}+1=\frac{2v}{v-1}$$

$$\therefore \int \frac{(v-1)dv}{2v}=\int dx$$

$$\therefore \int \frac{1}{2}dV-\int \frac{1}{2v}dv=\int dx=\frac{v}{2}-\frac{1}{2}\log|v|=x+c$$

$$\therefore \frac{x+y}{2}-\frac{1}{2}\log|x+y|=x+c$$

We have x $=\frac{2}{3}$, y $=\frac{1}{3}$

$$\therefore \frac{1}{2}-\frac{1}{2}\log|1|=\frac{2}{3}+c \Rightarrow c=\frac{1}{2}-\frac{2}{3}=\frac{-1}{6}$$

$$\therefore \frac{x+y}{2}-\frac{1}{2}\log|x+y|=x-\frac{1}{6}$$

$$\therefore \frac{x+y}{2}-\frac{1}{2}\log|x+y|=x-\frac{1}{6}$$

$$\therefore (x+y)-\log|x+y|=2x-\frac{2}{6} \Rightarrow y-x+\frac{1}{3}=\log|x+y|$$

Question183

If $y=\log_{10}x+\log_x 10+\log_x x+\log_{10}10$, then $\frac{dy}{dx}=$ MHT CET 2021 (23 Sep Shift 1)

Options:

A. $\frac{1}{x\log_e}+\frac{1}{x\log_{10}e}$

B. $\frac{1}{x\log_e 10}+\frac{\log_e 10}{x(\log_{10}e)^2}$

C. $\frac{1}{x\log_e 10}+\frac{1}{x\log_{10}e}$

D. $\frac{1}{x\log_e 10}+\frac{\log_0 10}{x(\log_e x)^2}$

Answer: D

Solution:

$$y=\frac{\log_e x}{\log_e 10}+\frac{\log_e 10}{\log_e x}+1+1$$

$$\therefore \frac{dy}{dx}=\left(\frac{1}{\log_e 10}\right)\left(\frac{1}{x}\right)+(\log_e 10)\left[\frac{-\frac{1}{x}}{(\log_e x)^2}\right]$$

$$=\frac{1}{x\log_e 10}-\frac{\log_e 10}{x(\log_e x)^2}$$

Question184

The differential equation of the family of parabolas with focus at the origin and the X - axis as axis, is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $-y\left(\frac{dy}{dx}\right)^2=2x\frac{dy}{dx}-y$

B. $y\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}+y=0$

C. $y\left(\frac{dy}{dx}\right)^2+4x\frac{dy}{dx}=4xy$

D. $y\left(\frac{dy}{dx}\right)^2+y=2xy\frac{dy}{dx}$

Answer: A

Solution:

Equation of parabola is $y^2=4a(x+a)$

$$2y\frac{dy}{dx}=4a\Rightarrow a=\frac{y\frac{dy}{dx}}{2}$$

Substituting value of ' a ', we get

$$y^2=4\left(\frac{y\frac{dy}{dx}}{2}\right)\left[x+\left(\frac{y\frac{dy}{dx}}{2}\right)\right]=\left(2y\frac{dy}{dx}\right)\left(\frac{2x+y\frac{dy}{dx}}{2}\right)$$

$$2y^2=\left(2y\frac{dy}{dx}\right)\left(2x+y\frac{dy}{dx}\right)=4xy\frac{dy}{dx}+2y^2\left(\frac{dy}{dx}\right)^2$$

$$\therefore \quad y=2x\frac{dy}{dx}+y\left(\frac{dy}{dx}\right)^2$$

Question185

The order of the differential equation whose solution is $\mathrm{y}=\mathrm{a}\cos\mathrm{x}+\mathrm{b}\sin\mathrm{x}+\mathrm{ce}^{-\mathrm{x}}$ is MHT CET 2021 (23 Sep Shift 1)

Options:

A. 3

B. 4

C. 2

D. 1

Answer: A

Solution:

$\mathrm{y}=\mathrm{a}\cos\mathrm{x}+\mathrm{b}\sin\mathrm{x}+\mathrm{ce}^{-\mathrm{x}}$ Since the equation has 3 arbitrary constants, the order of differential equation is 3 .

Question186

The general solution of the differential equation $(2y-1)dx-(2x+3)dy=0$ is MHT CET 2021 (23 Sep Shift 1)

Options:

A. $(2x+3)^2=c(2y-1)$

B. $\frac{2x+3}{2y-1}=c$

C. $(2x+3)(2y-1)=c$

D. $(2x+3)(2y-1)^2=c$

Answer: B

Solution:

$$(2y-1)dx-(2x+3)dy=0 \Rightarrow \frac{dy}{dx}=\frac{(2y-1)}{(2x+3)}$$

$$\therefore \int \frac{dy}{2y-1}=\int \frac{dx}{2x+3}$$

$$\frac{\log|2y-1|}{2}=\frac{\log|2x+3|}{2}+\log c_1 \Rightarrow \log\left|\frac{2x+3}{2y-1}\right|=-\log c_1=\log c$$

$$\therefore \frac{2x+3}{2y-1}=c$$

Question187

The general solution of $\frac{dy}{dx}=\frac{x+y}{x-y}$ is MHT CET 2021 (22 Sep Shift 2)

Options:

A. $\tan^{-1}\frac{x}{y}+\frac{1}{2}\log\left|x^2+y^2\right|=c$

B. $\tan^{-1}\frac{y}{x}+\frac{1}{2}\log\left|x^2+y^2\right|=c$

C. $\tan^{-1}\frac{y}{x}-\frac{1}{2}\log\left|x^2+y^2\right|=c$

D. $\tan^{-1}\frac{x}{y}-\frac{1}{2}\log\left|x^2+y^2\right|=c$

Answer: C

Solution:

$$\frac{dy}{dx} = \frac{x+y}{x-y}$$

Put $y = vx$

$$\therefore \frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\therefore v + x\frac{dv}{dx} = \frac{x+vx}{x-vx} = \frac{1+v}{1-v}$$

$$\therefore x\frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v-v+v^2}{1-v} = \frac{1+v^2}{1-v}$$

$$\therefore \int \frac{(1-v)}{1+v^2}dv = \int \frac{dx}{x}$$

$$\therefore \int \frac{1}{1+v^2}dv - \frac{1}{2}\int \frac{2v}{1+v^2}dv = \int \frac{dx}{x}$$

$$\therefore \tan^{-1}(v) - \frac{1}{2}\log\left|1+v^2\right| = \log|x| + c$$

$$\therefore \tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|1+\frac{y^2}{x^2}\right| - \frac{1}{2}\log\left|x^2\right| = c$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\left[\log\left|\frac{x^2+y^2}{x^2}\right| + \log\left|x^2\right|\right] = c$$

$$\tan^{-1}\left(\frac{y}{x}\right) - \frac{1}{2}\log\left|x^2+y^2\right| = c$$

Question188

The particular solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x^2-x}$, when $x = 2$ and $y = 1$ is MHT CET 2021 (22 Sep Shift 2)

Options:

A. $xy = 4x - 6$

B. $xy = 2x - 2$

C. $xy = x - 2$

D. $xy = -x + 4$

Answer: C

Solution:

$\frac{dy}{dx} = \frac{y+1}{x^2 - x}$

$\therefore \int \frac{dy}{y+1} = \int \frac{dx}{x(x-1)}$

$\therefore \int \frac{dy}{y+1} = \int \left[\frac{1}{x-1} - \frac{1}{x}\right] dx \Rightarrow \log|y+1| = \log|x-1| - \log|x| +$
$\log c$

We have $x = 2$ and $y = 1$

$\therefore \log|2| = \log|1| - \log|1| + \log c \Rightarrow c = 2$

$\therefore \log|y+1| = \log|x-1| - \log|x| + \log|2|$

$\log|y+1| = \log\left|\frac{2(x-1)}{x}\right|$

$\therefore y + 1 = \frac{2(x-1)}{x} \Rightarrow xy + x = 2x - 2 \Rightarrow xy = x - 2$

Question189

The degree of the differential equation whose solution is $y^2 = 8a(x+a)$, is MHT CET 2021 (22 Sep Shift 2)

Options:

A. 2

B. 1

C. 4

D. 3

Answer: A

Solution:

We have $y^2 = 8ax + 8a^2$Differentiating w.r.t. x, we get

$$2y\frac{dy}{dx} = 8a \quad \Rightarrow \quad a = \left(\frac{y}{4}\right)\frac{dy}{dx}$$

Substituting value of ' a ' in eq. (1), we get

$$y^2 = 8\left(\frac{y}{4}\right)\frac{dy}{dx}(x) + 8\left[\left(\frac{y}{4}\right)\left(\frac{dy}{dx}\right)\right]^2 = 2xy\frac{dy}{dx} + \left(\frac{y^2}{2}\right)\left(\frac{dy}{dx}\right)^2$$

$$\therefore 2y^2 = 4xy\left(\frac{dy}{dx}\right) + y^2\left(\frac{dy}{dx}\right)^2$$

Hence order $= 1$, degree $= 2$

Question190

The differential equation of all parabolas having vertex at the origin and axis along positive Y-axis is MHT CET 2021 (22 Sep Shift 2)

Options:

A. $x^2\frac{dy}{dx} - y = 0$

B. $x\frac{dy}{dx} + 2y = 0$

C. $x\frac{dy}{dx} + y = 0$

D. $2x\frac{dy}{dx} - y = 0$

Answer: D

Solution:

We have $y^2 = 4ax$

$$\therefore 2y\frac{dy}{dx} = 4a \quad \Rightarrow a = \left(\frac{y}{2}\right)\frac{dy}{dx}$$

$$\therefore y^2 = 4\left(\frac{y}{2}\right)\left(\frac{dy}{dx}\right)x \Rightarrow y^2 = 2xy\frac{dy}{dx}$$

$$\therefore 2x\frac{dy}{dx} - y = 0$$

Question191

The particular solution of the differential equation $y(1+\log x) = (\log x^x)\frac{dy}{dx}$, when $y(e) = e^2$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $2 \text{ ex} \log x - y = e^2$

B. $3ex \log yx - y = 2e^2$

C. $ex \log x + y = 2e^2$

D. $ex \log x - y = 0$

Answer: D

Solution:

$$y(1+\log x) = (\log x^x)\frac{dy}{dx}$$
$$\therefore \int \frac{(1+\log x)}{(\log x^x)} = \int \frac{dy}{y}$$

We know that $\frac{d}{dx}(\log x^x) = (1+\log x)dx$

$$\therefore \log|\log \mathrm{x}^{\mathrm{x}}| = \log|\mathrm{y}| + \log \mathrm{c}$$
$$\text{We have } \mathrm{y}(\mathrm{e}) = \mathrm{e}^2$$
$$\therefore \log|\log \mathrm{e}^{\mathrm{e}}| = \log|\mathrm{e}^2| + \log \mathrm{c}$$
$$\therefore \log|\mathrm{e}\log \mathrm{e}| = 2\log|\mathrm{e}| + \log \mathrm{c}$$
$$\therefore 1 = 2 + \log \mathrm{c} \Rightarrow \log \mathrm{c} = -1$$
$$\therefore \log|\log \mathrm{x}^{\mathrm{x}}| = \log|\mathrm{y}| - \log \mathrm{e}$$
$$\therefore \log|\mathrm{x}\log \mathrm{x}| + \log \mathrm{e} = \log|\mathrm{y}|$$
$$\therefore \mathrm{ex}\log \mathrm{x} - \mathrm{y} = 0$$

Question192

The differential equation of all family of lines $y = mx + \frac{4}{m}$ obtained by eliminating the arbitrary constant m is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $y\left(\frac{dy}{dx}\right) = 4$

B. $y\left(\frac{dy}{dx}\right)^2 + y\left(\frac{dy}{dx}\right) + 4 = 0$

C. $x\left(\frac{dy}{dx}\right) + 4 = 0$

D. $x\left(\frac{dy}{dx}\right)^2 - y\left(\frac{dy}{dx}\right) + 4 = 0$

Answer: D

Solution:

Solution

Given the family $y = mx + \frac{4}{m}$ with parameter m.

Differentiate w.r.t. x:

$$\frac{dy}{dx} = m.$$

Eliminate m by substituting $m = \frac{dy}{dx}$ into the original family:

$$y = x\frac{dy}{dx} + \frac{4}{\frac{dy}{dx}}.$$

Multiply by $\frac{dy}{dx}$:

$$\frac{dy}{dx}y = x\left(\frac{dy}{dx}\right)^2 + 4.$$

Bring all terms to one side:

$$\boxed{x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0}.$$

Question193

Solution of the differential equation $y' = \frac{(x^2+y^2)}{xy}$, where $y(1) = -2$ is given by MHT CET 2021 (22 Sep Shift 1)

Options:

A. $y^2 = 4x^2 \log x^2 + x^2$

B. $y^2 = x^2 \log x - x^2$

C. $y^2 = x \log x^2 + 4x^2$

D. $y^2 = x^2 \log x^2 + 4x^2$

Answer: D

Solution:

$$\frac{dy}{dx}=\frac{x^2+y^2}{xy}=\frac{x}{y}+\frac{y}{x}$$

Let $\frac{x}{y}=v \Rightarrow y=\frac{v}{x}$

$$\therefore \frac{dy}{dx}=\frac{v-x\left(\frac{dv}{dx}\right)}{v^2}$$

$$\therefore \frac{v-x\left(\frac{dv}{dx}\right)}{v^2}=v+\frac{1}{v} \Rightarrow \frac{1}{v}-\left(\frac{x}{v^2}\right)\frac{dv}{dx}=v+\frac{1}{v}$$

$$\therefore \int \frac{dv}{v^3}=-\int \frac{dx}{x}$$

$$\therefore \frac{-1}{2v^2}=-\log x-c \quad \Rightarrow \frac{1}{2v^2}=\log x+c$$

$$\therefore \frac{y^2}{2x^2}=\log \mathrm{x}+\mathrm{c} \quad \Rightarrow \mathrm{y}^2=\mathrm{x}^2\log \mathrm{x}^2+2\mathrm{x}^2\mathrm{c}$$

We have $\mathrm{y}(1)=-2$

$\therefore 4=0+2c \Rightarrow c=2$

$\therefore \mathrm{y}^2=\mathrm{x}^2\log \mathrm{x}^2+4\mathrm{x}^2$

Question194

The general solution of $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$ is MHT CET 2021 (22 Sep Shift 1)

Options:

A. $\tan(x+y)-\sec(x+y)=x^2+c$

B. $\tan(x+y)+\sec(x+y)=x^2+c$

C. $\tan(x+y)+\sec(x+y)=x+c$

D. $\tan(x+y)-\sec(x+y)=x+c$

Answer: D

Solution:

$$\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$$

$$\therefore \frac{dy}{dx}=\sin(x+y)$$

$$\text{Put } x+y=t \quad \Rightarrow 1+\frac{dy}{dx}=\frac{dt}{dx}$$

$$\therefore \frac{dt}{dx}-1=\sin t \Rightarrow \frac{dt}{dx}=1+\sin t$$

$$\therefore \int \frac{dt}{1+\sin t}=\int dx$$

$$\therefore \int \frac{(1-\sin t)dt}{1-\sin} t=\int dx \Rightarrow \int \frac{1-\sin t}{\cos^2 t} dt=\int dx$$

$$\Rightarrow \int\left(\sec^2 t-\operatorname{sect} \tan t\right) dt=\int dx$$

$$\therefore \tan t-\sec t=x+c$$

$$\therefore \tan(x+y)-\sec(x+y)=x+c$$

Question195

Bismuth has half life period of 5 days. A sample originally has a mass of $1000 \mathrm{mg}$, then the mass of Bismuth after 30 days is MHT CET 2021 (22 Sep Shift 1)

Options:

A. 16.625

B. 13.625

C. 14.625

D. 15.625

Answer: D

Solution:

Half life period of bismuth is 5 days.

Initial mass $= 1000\text{mg}$

$\therefore$ Mass left after 5 days $= 500\text{mg}$

Mass left after 10 days $= 250\text{mg}$

Mass left after 15 days $= 125\text{mg}$

Mass left after 20 days $= 62.5\text{mg}$

Mass left after 25 days $= 31.25\text{mg}$

Mass left after 30 days $= 15.625\text{mg}$

Question196

If the half life period of a substance is 5 years, then the total amount of the substance left after 15 years, when initial amount is 64gms is MHT CET 2021 (21 Sep Shift 2)

Options:

A. 8 gms

B. 16 gms

C. 2 gms

D. 32 gms

Answer: A

Solution:

Half life period $= 5$ years

Initial quantity of substance $= 64$ grams

$\therefore$ Quantity left after 5 years $= \frac{64}{2} = 32$ grams

$\therefore$ Quantity left after 10 years $= \frac{32}{2} = 16$ grams

$\therefore$ Quantity left after 15 years $= \frac{16}{2} = 8$ grams

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Question197

The general solution of the differential equation.

$$\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)dx-\left[\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right]dy=0 \text{ is}$$

MHT CET 2021 (21 Sep Shift 2)

Options:

A. $\mathrm{y}^2\sin\left(\frac{\mathrm{y}}{\mathrm{x}}\right)=\mathrm{k}$

B. $\mathrm{x}\sin\left(\frac{\mathrm{y}}{\mathrm{x}}\right)=\mathrm{k}$

C. $\sin\left(\frac{\mathrm{y}}{\mathrm{x}}\right)=\mathrm{k}$

D. $y\sin\left(\frac{y}{x}\right)=k$

Answer: D

Solution:

We have $\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)dx-\left[\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)\right]dy=0$

$$\therefore \frac{dy}{dx}=\frac{\left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right)}{\left(\frac{x}{y}\right)\sin\left(\frac{y}{x}\right)+\cos\left(\frac{y}{x}\right)}$$

Put $\frac{y}{x}=v\Rightarrow y=vx\Rightarrow\frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore \mathrm{v}+\mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}}=\frac{\mathrm{v}\cos\mathrm{v}}{\frac{1}{\mathrm{v}}\sin\mathrm{v}+\cos\mathrm{v}}=\frac{\mathrm{v}^2\cos\mathrm{v}}{\sin\mathrm{v}+\mathrm{v}\cos\mathrm{v}}\Rightarrow\mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-\mathrm{v}\sin\mathrm{v}}{\sin\mathrm{v}+\mathrm{v}\cos\mathrm{v}}$$

$$\therefore\int\frac{\sin\mathrm{v}+\mathrm{v}\cos\mathrm{v}}{\mathrm{v}\sin\mathrm{v}}\mathrm{dv}=\int\frac{-\mathrm{dx}}{\mathrm{x}}\Rightarrow\int\frac{1}{\mathrm{v}}\mathrm{dv}+\int\cot\mathrm{v}d\mathrm{v}=-\int\frac{\mathrm{dx}}{\mathrm{x}}$$

$$\therefore\log|\mathrm{v}|+\log|+\sin\mathrm{x}|=-\log|\mathrm{x}|+\log\mathrm{k}\Rightarrow\log|(\mathrm{v})(\sin\mathrm{v})(\mathrm{x})|$$
$$=\log\mathrm{k}$$

$$\therefore y\sin\left(\frac{y}{x}\right)=k$$

Question198

If m is order and n is degree of the differential equation

$\left(\frac{d^2y}{dx^2}\right)^5+4\frac{\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{d^3y}{dx^3}\right)}+\left(\frac{d^3y}{dx^3}\right)=x^2-1$**, then MHT CET 2021 (21 Sep Shift 2)**

Options:

A. m=3, n=1

B. m=3, n=2

C. m=3, n=3

D. m=3, n=5

Answer: B

Solution:

$$\text{We have } \left(\frac{d^2y}{dx^2}\right)^5 + \frac{4\left(\frac{d^2y}{dx^2}\right)}{\left(\frac{d^3y}{dx^3}\right)} + \left(\frac{d^3y}{dx^3}\right) = x^2 - 1$$

$$\therefore \left(\frac{d^3y}{dx^3}\right)\left(\frac{d^2y}{dx^2}\right)^5 + 4\left(\frac{d^2y}{dx^2}\right)\left(\frac{d^3y}{dx^3}\right)^2 = \left(x^2 - 1\right)\left(\frac{d^3y}{dx^3}\right)$$

$$\therefore \text{ order } = 3, \text{degree } = 2$$

Question199

The differential equation of the family of circles touching y-axis at the origin is MHT CET 2021 (21 Sep Shift 2)

Options:

A. $x^2 - y^2 - 2xy\frac{dy}{dx} = 0$

B. $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

C. $x^2 + y^2 - 2xy\frac{dy}{dx} = 0$

D. $x^2 + y^2 + 2xy\frac{dy}{dx} = 0$

Answer: B

Solution:

Since circles touch Y axis at origin, the centres of the circles lie on X axis.Let centre be $(h, 0)$ and radius $= h$

$$\therefore (x - h)^2 + (y - 0)^2 = h^2 \Rightarrow x^2 - 2hx + y^2 = 0$$

Differentiating w.r.t. x, we get

$$2x-2h+2y\frac{dy}{dx}=0 \quad \Rightarrow x+y\frac{dy}{dx}=h$$

Substituting value of h in eq. (1), we get

$$x^2-2\left(x+y\frac{dy}{dx}\right)x+y^2=0$$

$$\therefore x^2-2x^2-2xy\frac{dy}{dx}+y^2=0 \quad \Rightarrow x^2-y^2+2xy\frac{dy}{dx}=0$$

Question200

$I: y'=\frac{y+x}{x}$; $\quad$ II $: y'=\frac{x^2+y}{x^3}$; $\quad$ III $: y'=\frac{2xy}{y^2-x^2}$

S1: Differential equations given by I and II are homogeneous differential equations.

S2: Differential equations given by II and III are homogeneous differential equations.

S3: Differential equations given by I and III are homogeneous differential equations.

Options:

A. only S1 is valid

B. both S1 and S2 are valid

C. only S3 is valid

D. only S2 is valid

Answer: C

Solution:

We will check all 3 equations.

I : $\frac{dy}{dx}=\frac{y+x}{x} \Rightarrow$ All terms have same degree equal to 1 .

II : $\frac{dy}{dx}=\frac{x^2+y}{y^3} \Rightarrow$ Here all terms have different degrees.,

III : $\frac{dy}{dx}=\frac{2xy}{y^2-x^2} \Rightarrow$ All terms have same degree equal to 2 .

Question201

The general solution of the differential equation $\frac{dy}{dx} = \frac{x+2y-1}{x+2y+1}$ is MHT CET 2021 (21 Sep Shift 1)

Options:

A. $3(x+y) + 4\log|3x+6y-1| = K$

B. $3(\mathrm{x}-\mathrm{y}) + 4\log|3\mathrm{x}+6\mathrm{y}-1| = \mathrm{K}$

C. $6(-x+y) + 4\log|3x+6y-1| = K$

D. $6(\mathrm{x}+\mathrm{y}) + 4\log|3\mathrm{x}+6\mathrm{y}-1| = \mathrm{K}$

Answer: C

Solution:

$$\frac{dy}{dx} = \frac{x+2y-1}{x+2y+1}$$

Put $x+2y = t \Rightarrow x+2y = t \Rightarrow 1+2\frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = \frac{\left(\frac{dt}{dx}-1\right)}{2}$

$$\therefore \frac{\left(\frac{dt}{dx}-1\right)}{2} = \frac{t-1}{t+1} \Rightarrow \frac{dt}{dx} - 1 = \frac{2t-2}{t+1}$$
$$\therefore \frac{dt}{dx} = \frac{2t-2}{t+1} + 1 = \frac{3t-1}{t+1}$$

$$\therefore \int \frac{t+1}{3t-1} dt = \int dx$$
$$\therefore \frac{1}{3}\int \frac{3(t+1)}{3t-1} dt = \int dx \Rightarrow \frac{1}{3}\int \frac{3t-1+4}{3t-1} dt = \int dx$$
$$\therefore \frac{1}{3}\int dt + \frac{4}{3}\int \frac{dt}{3t-1} = \int dx$$
$$\therefore \frac{t}{3} + \frac{4}{3}\frac{\log|3t-1|}{3} = x + c_1$$
$$\therefore \frac{x+2y}{3} + \frac{4}{3}\frac{\log|3(x+2y)-1|}{3} = x + c_1$$
$$\therefore 3x+6y+4\log|3x+6y-1| = 9x+9c_1$$
$$\therefore 6(-x+y) + 4\log|3x+6y-1| = K$$

Question202

The differential equation of family of circles whose centre lie on X-axis is MHT CET 2021 (21 Sep Shift 1)

Options:

A. $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$

B. $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2+1=0$

C. $y\left(\frac{d^2y}{dx^2}\right)-\left(\frac{dy}{dx}\right)^2-1=0$

D. $y\left(\frac{d^2y}{dx^2}\right)+\left(\frac{dy}{dx}\right)^2-1=0$

Answer: B

Solution:

Let $(h, 0)$ be the centre of the circle and ' r ' be the radius.

$$\therefore (\mathrm{x}-\mathrm{h})^2+(\mathrm{y}-0)^2=\mathrm{r}^2 \Rightarrow (\mathrm{x}-\mathrm{h})^2+\mathrm{y}^2=\mathrm{r}^2$$

Differentiating w.r.t. x, we get

$$\therefore 2(\mathrm{x}-\mathrm{h})(1)+2\mathrm{y}\frac{\mathrm{dy}}{\mathrm{dx}}=0 \quad \Rightarrow \mathrm{h}=\mathrm{x}+\mathrm{y}\frac{\mathrm{dy}}{\mathrm{dx}}$$

Substituting value of ' h ' in eq. (1), we get

$$y^2\left(\frac{dy}{dx}\right)^2+y^2=r^2 \Rightarrow y^2\left[1+\left(\frac{dy}{dx}\right)^2\right]=r^2$$

Differentiating w.r.t. x, we get

$$y^2\left(2\frac{dy}{dx}\cdot\frac{d^2y}{dx^2}\right)+\left[1+\left(\frac{dy}{dx}\right)^2\right]2y\frac{dy}{dx}=0$$

$$\therefore y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$$

Question203

The general solution of the differential equation

$$y(1+\log x)\left(\frac{dx}{dy}\right)$$

$-x \log x=0$ **is MHT CET 2021 (21 Sep Shift 1)**

Options:

A. $\mathrm{y}(1+\log \mathrm{x})=\mathrm{c}$

B. $\mathrm{x} \log \mathrm{x}=\mathrm{yc}$

C. $x \log x=y+c$

D. $\log \mathrm{x}-\mathrm{y}=\mathrm{c}$

Answer: B

Solution:

$$\begin{aligned}
& y(1+\log x)\left(\frac{dy}{dx}\right)-x \log x=0 \\
& \therefore \int \frac{(1+\log x)}{x \log x} dx=\int \frac{dy}{y} \\
& \int \frac{dx}{x \log x}+\int \frac{dx}{x}=\int \frac{dy}{y} \\
& \therefore \log (\log x)+\log x=\log y+\log c \\
& \therefore \log [x \log x]=\log (y, c) \Rightarrow x \log x=y c
\end{aligned}$$

Question204

The general solution of the differential equation $\left(3xy+y^2\right) dx+\left(x^2+xy\right) dy=0$ is MHT CET 2021 (21 Sep Shift 1)

Options:

A. $x^2\left(2xy-y^2\right)=c$

B. $x^2\left(y^2-2xy\right)=c$

C. $x\left(2xy+y^2\right)=c$

D. $\mathrm{x}^2\left(2\mathrm{xy}+\mathrm{y}^2\right)=\mathrm{c}$

Answer: D

Solution:

$$\left(3xy+y^2\right)dx+\left(x^2+xy\right)dy=0$$
$$\therefore \frac{dy}{dx}=-\frac{\left(3xy+y^2\right)}{\left(x^2+xy\right)}$$

Put $y=vx \Rightarrow \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$\therefore v+x\frac{dv}{dx}=-\frac{\left(3vx^2+v^2x^2\right)}{\left(x^2+vx^2\right)}=-\frac{3v+v^2}{1+v}$$
$$\therefore x\frac{dv}{dx}=-\frac{3v+v^2}{1+v}-v=-\frac{\left(2v^2+4v\right)}{1+v}$$

$$\therefore \int\frac{1+v}{\left(2v^2+4v\right)}dv=\int\frac{-1}{x}dx$$
$$\therefore \frac{1}{2}\int\frac{v+1}{v^2+2v}dv=-\int\frac{dx}{x} \Rightarrow \frac{1}{4}\int\frac{2(v+1)}{v^2+2v}dv=-\log x+\log c_1$$

$$\therefore \frac{1}{4}\log\left(v^2+2v\right)+\log x=\log c_1 \Rightarrow \frac{1}{4}\log\left(\frac{y^2}{x^2}+\frac{2y}{x}\right)+\log x=\log c_1$$
$$\therefore \frac{1}{4}\log\left(\frac{y^2+2xy}{x^2}\right)+\log x=\log c_1 \Rightarrow \log\left(\frac{y^2+2xy}{x^2}\right)+\log x^4=4\log c_1$$
$$\therefore \log\left[\left(\frac{y^2+2xy}{x^2}\right)\left(x^4\right)\right]=\log c_1^4 \Rightarrow \log\left[x^2\left(y^2+2xy\right)\right]=\log c_1^4$$
$$\therefore x^2\left(y^2+2xy\right)=c$$

Question205

A body at an unknow temperature is placed in a room which is held at a constant temperature of 30°F. If after 10 minutes the temperature of the body is 0°F and after 20 minutes the temperature of the body is 15°F, then the expression for the temperature of the body at any time t is MHT CET 2021 (21 Sep Shift 1)

Options:

A. $T=-60e^{-0.069t}-30$

B. $T=-60e^{-0.03010t}+30$

C. $T=60e^{-0.069t}+30$

D. $T=60e^{-0.069t}-30$

Answer: B

Solution:

We have $\frac{dT}{dt} \propto (30 - T)$

$$\therefore \frac{dT}{dt} = -K(30 - T) \Rightarrow \int \frac{dT}{30 - T} = \int -Kt$$
$$\therefore \log|30 - T| = -kt + c$$

From given data, we write

$$\log|30 - 0| = -10K + c$$
$$\log|30 - 15| = -20K + c$$

Solving (2) and (3), we get

$$\log\left(\frac{30}{15}\right) = 10\ K \Rightarrow K = \frac{1}{10}\log 2$$

Substituting value of K in eq. (2), we get

$$\log 30 = (-10)\left(\frac{\log 2}{10}\right) + c \Rightarrow c = \log 60$$

Thus eq. (1) becomes

$$\log|30 - T| = \frac{-\log 2}{10}t + \log 60$$
$$\therefore \log\left|\frac{30 - T}{60}\right| = \frac{-0.3010}{10}t = -0.03010t$$
$$\therefore \frac{30 - T}{60} = e^{-0.03010t} \quad \therefore T = -60e^{-0.03010t} + 30$$

Question206

The population of a city increases at a rate proportional to the population at that time. If the population of the city increase from 20 lakhs to 40 lakhs in 30 years, then after another 15 years the population is MHT CET 2021 (20 Sep Shift 2)

Options:

A. $10\sqrt{2}$ lakhs

B. $40\sqrt{2}$ lakh

C. $30\sqrt{2}$ lakhs

D. None of these

Answer: B

Solution:

We have $\frac{\mathrm{dP}}{\mathrm{dt}} \propto \mathrm{P} \Rightarrow \frac{\mathrm{dP}}{\mathrm{dt}} = \mathrm{kP}$

$$\therefore \int \frac{\mathrm{dP}}{\mathrm{dt}} = \int \mathrm{k}dt \Rightarrow \log \mathrm{P} = \mathrm{kt} + \mathrm{c}$$

From given data, we write

$$\begin{aligned}
&\log 20 = \mathrm{k}(0) + \mathrm{c} \Rightarrow \mathrm{c} = \log 20 \\
&\therefore \log \mathrm{P} = \mathrm{kt} + \log 20 \\
&\text{Also } \log 40 = 30\mathrm{k} + \log 20 \\
&\therefore \log 40 - \log 20 = 30\mathrm{k} \Rightarrow \mathrm{k} = \frac{1}{30}\log 2 \\
&\therefore \log \mathrm{P} = \left(\frac{\log 2}{30}\right)\mathrm{t} + \log 20 \\
&\text{When } \mathrm{t} = 30 + 15 = 45 \\
&\therefore \log \mathrm{P} = \left(\frac{\log 2}{30}\right)(45) + \log 20 = (\log 2)\left(\frac{3}{2}\right) + \log 20 \\
&= \log(2)^{\frac{3}{2}} + \log 20 = \log(2\sqrt{2} \times 20) \\
&\therefore \mathrm{P} = 40\sqrt{2} \text{ lakhs}
\end{aligned}$$

Question207

The general solution of $\left(x\frac{dy}{dx} - y\right) \sin \frac{y}{x} = x^3 e^x$ **is MHT CET 2021 (20 Sep Shift 2)**

Options:

A. $\mathrm{e}^{\mathrm{x}}(\mathrm{x}-1) + \cos \frac{\mathrm{y}}{\mathrm{x}} + \mathrm{c} = 0$

B. $\mathrm{xe}^{\mathrm{x}} + \cos \frac{\mathrm{y}}{\mathrm{x}} + \mathrm{c} = 0$

C. $\mathrm{e}^{\mathrm{x}}(\mathrm{x}+1) + \cos \frac{\mathrm{y}}{\mathrm{x}} + \mathrm{c} = 0$

D. $\mathrm{ex}^{\mathrm{x}} - \cos \frac{\mathrm{y}}{\mathrm{x}} + \mathrm{c} = 0$

Answer: A

Solution:

Given differential equation:

$$\left(x\frac{dy}{dx} - y\right)\sin\frac{y}{x} = x^3e^x$$

Step 1: Substitute $y = vx$

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Step 2: Substitute in the equation

$$\left(x(v + x\frac{dv}{dx}) - vx\right)\sin\frac{y}{x} = x^3e^x$$

Simplify:

$$(x^2\frac{dv}{dx})\sin v = x^3e^x$$

Step 3: Simplify

$$\frac{dv}{dx} = xe^x\csc v$$

Step 4: Separate variables

$$\sin v\,dv = xe^x\,dx$$

Step 5: Integrate both sides

$$\int\sin v\,dv = \int xe^x\,dx$$

$$-\cos v = e^x(x-1) + C$$

Step 6: Substitute back $v = \frac{y}{x}$

$$-\cos\frac{y}{x} = e^x(x-1) + C$$

$$\Rightarrow e^x(x-1) + \cos\frac{y}{x} + C = 0$$

✅ **Final Answer:**

$$\boxed{e^x(x-1) + \cos\frac{y}{x} + c = 0}$$

Question208

The differential equation of an ellipse whose major axis is twice its minor axis, is MHT CET 2021 (20 Sep Shift 2)

Options:

A. $x+4y\frac{dy}{dx}=0$

B. $x-4y\frac{dy}{dx}=0$

C. $x+2y\frac{dy}{dx}=0$

D. None of these

Answer: A

Solution:

For the given ellipse, we have $\mathrm{a}=2\,\mathrm{b}$

$$\therefore \because \frac{\mathrm{x}^2}{4\,\mathrm{b}^2}+\frac{\mathrm{y}^2}{\mathrm{b}^2}=1 \Rightarrow \mathrm{x}^2+4\mathrm{y}^2=4\,\mathrm{b}^2$$

Differentiating both sides w.r.t., we get

$$2x+8y\frac{dy}{dx}=0 \Rightarrow x+4y\frac{dy}{dx}=0$$

Question209

The general solution of the differential equation $\cos x\cdot\sin y dx+\sin x\cdot\cos y dy=0$ **is MHT CET 2021 (20 Sep Shift 2)**

Options:

A. $\sin x+\sin y=c$

B. $\cos x+\cos y=c$

C. $\sin x\cdot\sin y=c$

D. $\cos x\cdot\cos y=c$

Answer: C

Solution:

$$\cos x \sin y dx + \sin x \cos y dy = 0$$

$$\therefore \cos x \sin y dx = -\sin x \cos y dy$$

$$\therefore \int \frac{\cos x}{\sin x} dx = -\int \frac{\cos y}{\sin y} dy$$

$$\therefore \log|\sin x| = -\log|\sin y| + c_1$$

$$\therefore \log|\sin x| + \log|\sin y| = c_1 \Rightarrow \log[\sin x \sin y] = c_1$$

$$\therefore \sin x \sin y = e^{c_1} = c$$

Question210

If m is order and n is degree of the differential equation $y = \frac{dp}{dx} + \sqrt{a^2p^2 - b^2}$, where $p = \frac{dp}{dx}$, then the value of $m + n$ is MHT CET 2021 (20 Sep Shift 2)

Options:

A. 2

B. 3

C. 4

D. 5

Answer: C

Solution:

Given:

$$y=\frac{dp}{dx}+\sqrt{a^2p^2-b^2}$$

and $p=\frac{dy}{dx}$

Step 1: Substitute $p=\frac{dy}{dx}$

Then,

$$\frac{dp}{dx}=\frac{d^2y}{dx^2}$$

So the equation becomes:

$$y=\frac{d^2y}{dx^2}+\sqrt{a^2\left(\frac{dy}{dx}\right)^2-b^2}$$

Step 2: Remove the square root

To find the **degree**, the equation must be free from radicals and fractions of derivatives.

Subtract $\frac{d^2y}{dx^2}$ from both sides:

$$y-\frac{d^2y}{dx^2}=\sqrt{a^2\left(\frac{dy}{dx}\right)^2-b^2}$$

Now square both sides:

$$\left(y-\frac{d^2y}{dx^2}\right)^2=a^2\left(\frac{dy}{dx}\right)^2-b^2$$

Step 3: Identify Order and Degree

- The **highest derivative** present is $\frac{d^2y}{dx^2}$ →
  Order (m) = 2
- The **power of the highest derivative** (after removing radicals) is 2 →
  Degree (n) = 2

Step 4: Find $m+n$

$$m+n=2+2=4$$

✅ **Final Answer:**

$$\boxed{m+n=4}$$

Question211

An ice ball melts at the rate which is proportional to the amount of ice at that instant. Half the quantity of ice melts in 20 minutes, x_0 is the initial quantity of ice. If after 40 minutes the amount of ice left is Kx_0, then $K=$ MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\frac{1}{2}$

B. $\frac{1}{8}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

Answer: C

Solution:

Half the quantity of ice melts in 20 minutes and x_0 is the initial quantity of ice.

$\therefore$ Quantity after 20 minutes $= \frac{x_0}{2}$

Quantity after 40 minutes $= \frac{1}{2}\left(\frac{x_0}{2}\right) = \frac{x_0}{4}$

Question212

The general solution of the differential equation $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ is given by MHT CET 2021 (20 Sep Shift 1)

Options:

A. $y = x\log(x+y) + c$

B. $x - y = \log(x+y) + c$

C. $x + y = \log(x+y) + c$

D. $y = x + \log(x+y) + c$

Answer: D

Solution:

$$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$$

Put $x + y = u \Rightarrow 1 + \frac{dy}{dx} = \frac{du}{dx}$

$$\therefore \frac{du}{dx} - 1 = \frac{u+1}{u-1} \Rightarrow \frac{du}{dx} = \frac{du+1}{dx-1} + 1 = \frac{2u}{u-1}$$
$$\therefore \left(\frac{u-1}{u}\right) du = 2dx$$

Integrating both sides, we get

$$\int du-\int \frac{du}{u}=\int 2dx$$
$$\therefore u-\log u=2x+c$$
$$\therefore x+y-\log(x+y)=2x+c \Rightarrow y=x+\log(x+y)+c$$

Question213

The differential equation of all circles which pass through the origin and whose centre lie on Y-axis is MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$

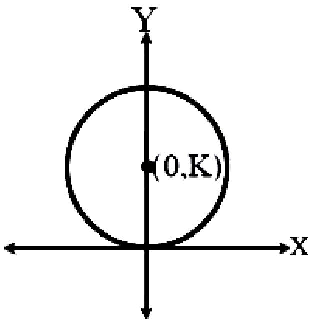
B. $\left(x^2+y^2\right)\frac{dy}{dx}-2xy=0$

C. $\left(x^2+y^2\right)\frac{dy}{dx}+2xy=0$

D. $\left(x^2+y^2\right)\frac{dy}{dx}+2xy=0$

Answer: A

Solution:



Let (0, K) be the centre of the circle.
Since the crcle passes through origin, we write

$(x-0)^2+(y-K)^2=K^2$

$\therefore x^2+y^2-2Ky+K^2=K^2$

$\therefore x^2+y^2-2Ky=0 \quad \ldots(1)$

Differentiating w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2k\frac{dy}{dx}=0$$

$\therefore K=\dfrac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}$ and substituting value of K in (1), we get

$$x^2+y^2-2\left(\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}\right)y=0$$

$$\therefore (x^2+y^2)\frac{dy}{dx}-2xy-2y^2\frac{dy}{dx}=0 \Rightarrow (x^2-y^2)\frac{dy}{dx}-2xy=0$$

Question214

A differential equation for the temperature ' T ' of a hot body as a function of time, when it is placed in a both which is held at a constant temperature of $32^{\circ}\mathrm{F}$, is given by (where k is a constant of proportionality) MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\frac{dT}{dt}=kT-32$

B. $\frac{dT}{dt}=kT+32$

C. $\frac{dT}{dt}=k(T-32)$

D. $\frac{dT}{dt}=32kT$

Answer: C

Solution:

The temperature T of the body will decrease with time. The body is kept in a bath of temperature 32°F.

$$\therefore \frac{\mathrm{dT}}{\mathrm{dt}} \alpha-(\mathrm{T}-32)$$
$$\Rightarrow \frac{\mathrm{dT}}{\mathrm{dt}}=-\mathrm{k}(\mathrm{T}-32)$$

Question215

The general solution of the differential equation $x+y\frac{dy}{dx}=\sec\left(x^2+y^2\right)$ is MHT CET 2021 (20 Sep Shift 1)

Options:

A. $\sin\left(x^2+y^2\right)=2x+c$

B. $\sin\left(x^2+y^2\right)+2x=c$

C. $\sin\left(x^2+y^2\right)+x=c$

D. $\cos\left(x^2+y^2\right)=2x+c$

Answer: A

Solution:

$$\text{We have, } x+y\frac{dy}{dx}=\sec\left(x^2+y^2\right)$$
$$\text{Put } x^2+y^2=u \Rightarrow 2x+2y\frac{dy}{dx}=\frac{du}{dx}$$
$$\therefore x+y\frac{dy}{dx}=\frac{1}{2}\frac{du}{dx}$$
$$\therefore \frac{1}{2}\frac{du}{dx}=\sec u$$
$$\therefore \int\frac{du}{\sec u}=\int 2dx$$
$$\therefore \sin u=2x+c$$
$$\Rightarrow \sin\left(x^2+y^2\right)=2x+c$$

Question216

The integrating factor of the differential equation $\frac{dy}{dx}(x \log x) + y = 4 \log x$ is MHT CET 2020 (20 Oct Shift 2)

Options:

A. $\log(\log x)$

B. x

C. e^x

D. $\log x$

Answer: D

Solution:

We have $\frac{dy}{dx}(x \log x) + y \quad = 4 \log x$

$\therefore \frac{dy}{dx} + \left(\frac{1}{x \log x}\right) y = \frac{4}{x}$

$\therefore \text{I.F.} = e^{\int \frac{dx}{x \log x}} = e^{\log(\log x)} = \log x$

Question217

The population of a village increases at a rate proportional to the population at that time. In a period of 10 years the population grew from 20,000 to 40,000 , then the population after another 20 years is MHT CET 2020 (20 Oct Shift 2)

Options:

A. $1,20,000$

B. $1,60,000$

C. $1,00,000$

D. 80,000

Answer: B

Solution:

We have $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = kP \quad \therefore \int \frac{dP}{P} = \int kdt \Rightarrow \log P = kt + \log c$

We have: when $t = 0, P = 20000$ and when $t = 10, P = 40000$

$\therefore \quad \log 20000 = \log c$

Now $\log 40000 = 10k + \log 20000$

$\therefore \log\left(\frac{40000}{20000}\right) = 10k \Rightarrow k = \frac{1}{10}\log 2$

$\therefore \log P = \left(\frac{1}{10}\log 2\right) t + \log 20000$

When $t = 30$, we write

$\log P = \frac{30}{10}\log 2 + \log 20000 = \log[(8)(20000)] \therefore P = 160000$

This problem can also be solved as follows :

From given data, we say that population doubles in 10 years.

$\therefore$ In next 10 years, population changes from 40,000 to 80,000 .

In 10 years after that, population changes from 80000 to 160000

Question218

Bismath has half life of 5 days. If sample originally has a mass of 800mg. then the mass remaining after 30 days will be MHT CET 2020 (20 Oct Shift 2)

Options:

A. 10mg.

B. 10.5mg.

C. 12mg.

D. 12.5mg.

Answer: D

Solution:

Original mass $= 800\text{mg}$.

Half life $= 5$ days.

We have to calculate remaining mass after 30 days. Understand that $30 \div 5 = 6$.

$$\therefore \text{ Mass remaining } = \left(\frac{1}{2}\right)^6 \times 800$$
$$= \frac{800}{64} = \frac{100}{8} = 12.5$$

Question219

The solution of the differential equation $x \cdot \sin\left(\frac{y}{x}\right) dy = \left[y \cdot \sin\left(\frac{y}{x}\right) - x\right] dx$ is MHT CET 2020 (20 Oct Shift 2)

Options:

A. $\cos\left(\frac{x}{y}\right) = \log|x| + c$

B. $\cos\left(\frac{y}{x}\right) = \log|y| + c$

C. $\cos\left(\frac{y}{x}\right) = \log|x| + c$

D. $\cos\left(\frac{x}{y}\right) = \log|y| + c$

Answer: C

Solution:

We have $x \cdot \sin\left(\frac{y}{x}\right) dy = \left[y \cdot \sin\left(\frac{y}{x}\right) - x\right] dx$

$$\therefore \frac{dy}{dx} = \frac{y \cdot \sin\left(\frac{y}{x}\right) - x}{x \sin\left(\frac{y}{x}\right)} = \left(\frac{y}{x}\right) - \frac{1}{\sin\left(\frac{y}{x}\right)}$$

$$\therefore \frac{dy}{dx} - \left(\frac{1}{x}\right) y = -\operatorname{cosec}\left(\frac{y}{x}\right)$$

Put $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore \left(\mathrm{v} + \frac{\mathrm{xdv}}{\mathrm{dx}}\right) - \mathrm{v} = -\operatorname{cosec} \mathrm{v} \Rightarrow \mathrm{x}\frac{\mathrm{dv}}{\mathrm{dx}} = -\operatorname{cosec} \mathrm{v}$$

$$\therefore \int \frac{\mathrm{dv}}{\operatorname{cosec} \mathrm{v}} = -\int \frac{\mathrm{dx}}{\mathrm{x}} \Rightarrow \int \sin \mathrm{vdv} = -\log|\mathrm{x}|$$

$$-\cos v = -\log|x| + c_1 \Rightarrow \log x + c = \cos v$$

$$\therefore \cos\left(\frac{y}{x}\right) = \log|x| + c$$

Question220

The order and the degree of the differential equation $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}}=7\left(\frac{d^2y}{dx^2}\right)$ are respectively MHT CET 2020 (20 Oct Shift 2)

Options:

A. 2,3

B. 3,3

C. 2,2

D. 3,2

Answer: A

Solution:

We have $\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}}=7\left(\frac{d^2y}{dx^2}\right)$ Raising both sides to power of 3 , we get

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^7=(7)^3\left(\frac{d^2y}{dx^2}\right)^3$$

Hence order $=2$, degree $=3$

Question221

The order and degree of the differential equation $y=px+\sqrt{a^2\mathbf{p}^2+b^2}$, where p $=\frac{\mathrm{d}y}{\mathrm{d}x}$ are respectively MHT CET 2020 (20 Oct Shift 1)

Options:

A. $1,2$

B. $3,1$

C. 2,1

D. $1,3$

Answer: A

Solution:

Given

$$y = px + \sqrt{a^2p^2 + b^2}$$
$$\therefore y - px = \sqrt{a^2p^2 + b^2}$$

On squaring both side we get

$$y^2 - 2pxy + p^2x^2 = a^2p^2 + b^2$$

$$y^2 - 2xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2 x^2 = a^2\left(\frac{dy}{dx}\right)^2 + b^2 \quad \ldots \left[p = \frac{dy}{dx}, \text{ given}\right]$$

Thus order is 1 and degree is 2 .

Question222

The equation of the curve whose slope at any point is equal to $2xy$ and which passes through the point $(0, 1)$ is MHT CET 2020 (20 Oct Shift 1)

Options:

A. $\cdot \log y = x^2$

B. $\log y = \frac{1}{x}$

C. $\frac{1}{y} = x$

D. $\log y = x$

Answer: A

Solution:

We have $\frac{dy}{dx} = 2xy$

$\therefore \int \frac{dy}{y} = \int 2xdx$

$\Rightarrow \log y = \frac{2x^2}{2} + C$

$\log y = x^2 + C$ and curve passes through point $(0, 1)$.

At $x = 0, y = 1$, we get $C = 0$

$\therefore \log y = x^2$

Question223

The rate of increase of population of a country is proportional to the number present. If the population doubles in 50 years, then the time taken by it to become four times of it self is MHT CET 2020 (20 Oct Shift 1)

Options:

A. 300 years

B. 100 years

C. 200 years

D. 400 years

Answer: B

Solution:

We have $\frac{dp}{dt} \propto p \Rightarrow \frac{dp}{dt} = kp \Rightarrow \int \frac{dp}{p} = \int kdt$

$\therefore \log p = kt + c \ldots (1)$

When $t = 0, p = p_0$ (initial population) $\Rightarrow c = \log p_0$

$\therefore \log\left(\frac{p}{p_0}\right) = kt \ldots (2)$

When $t = 50, \quad p = 2p_0$, we get

$\log 2 = 50k \Rightarrow k = \frac{1}{50}\log 2$

$\therefore \log\left(\frac{p}{p_0}\right) = \frac{t}{50}\log 2$

When $p = 4P_0$

$\log 4 = \frac{t}{50} \cdot \log 2 \Rightarrow 2\log 2 = \frac{t}{50}\log 2 \Rightarrow t = 100$ years

This problem can also be solved as follows :

Let initial population $= p$

Population doubles in 50 years

$\therefore$ After 50 years, population $= 2p$

After 100 years, population $= 4p$

Question224

The solution of the differential equation $\sin^{-1}\left(\frac{dy}{dx}\right) = x + y$ is MHT CET 2020 (20 Oct Shift 1)

Options:

A. $x=\tan(x+y)\cdot\sec(x+y)+c$

B. $x=\tan(x+y)-\sec(x+y)+c$

C. $x=\tan(x+y)+\sec(x+y)+c$

D. $x=\tan x\cdot\tan y+c$

Answer: B

Solution:

We have $\sin^{-1}\left(\frac{dy}{dx}\right)=x+y$

$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}}=\sin(\mathrm{x}+\mathrm{y})$$

Put $x+y=t \Rightarrow y=t-x \Rightarrow \frac{dy}{dx}=\frac{dt}{dx}-1$

$$\therefore \frac{\mathrm{dt}}{\mathrm{dx}}=1+\sin\mathrm{t} \Rightarrow \int\frac{\mathrm{dt}}{1+\sin\mathrm{t}}=\int\mathrm{dx}$$

$\int\frac{(1-\sin t)}{(1+\sin t)(1-\sin t)}dt=\int dx \Rightarrow x=\int\frac{1-\sin t}{1-\sin^2 t}dt$

$x=\int\frac{1-\sin t}{\cos^2 t}dt=\int\left(\sec^2 t-\sec t\tan t\right)dt$

$x=\tan t-\sec t+c$

$x=\tan(x+y)-\sec(x+y)+c$

Question225

The integrating factor of the differential equation y $\log_y\left(\frac{\mathrm{d}x}{\mathrm{dy}}\right)+x-\log\mathrm{y}=0$is MHT CET 2020 (20 Oct Shift 1)

Options:

A. $\log(\log y)$

B. $\log y$

C. y

D. e^y

Answer: B

Solution:

We have $y \log y\left(\frac{dx}{dy}\right)+x=\log y$

$\therefore \frac{\mathrm{dx}}{\mathrm{dy}}+\frac{\mathrm{x}}{\mathrm{y} \log \mathrm{y}}=\frac{1}{\mathrm{y}}$

$\therefore$ I.F. $=e^{\int \frac{1}{y \log y} dy}=e^{\log (\log y)}=\log y$

Question226

The order and degree of the differential equation $\left[1+\frac{1}{\left(\frac{dy}{dx}\right)^2}\right]^{\frac{5}{3}}=5 \frac{d^2 y}{dx^2}$ arerespectively

MHT CET 2020 (19 Oct Shift 2)

Options:

A. 2,3

B. 3,2

C. 5,2

D. 2,5

Answer: A

Solution:

Raising both sides of differential equation to degree 3 , we get

$$\left[1+\frac{1}{\left(\frac{dy}{dx}\right)^2}\right]^5=125\left(\frac{d^2 y}{dx^2}\right)^3$$
$$\left[\left(\frac{dy}{dx}\right)^2+1\right]^5=125 \cdot\left(\frac{d^2 y}{dx^2}\right)^3\left[\left(\frac{dy}{dx}\right)^2\right]^5$$

$\therefore$ Its order is 2 and degree is 3.

Question227

If $x=a \sin t-b \cos t$

$y=a \cos t+b \sin t$

then $y^3 \frac{d^2 y}{dx^2}+x^2+y^2=$

MHT CET 2020 (19 Oct Shift 2)

Options:

A. 0

B. 2

C. 1

D. -1

Answer: A

Solution:

We have, $x = a\sin t - b\cos t$ and $y = a\cos t + b\sin t$

$$x^2 + y^2 = \{[a\sin t - b\cos t]^2 + [a\cos t + b\sin t]^2\}$$
$$x^2 + y^2 = \{a^2 \cdot \sin^2 t - 2ab\sin t \cdot \cos t + b^2\cos^2 t + a^2\cos^2 t + 2ab\sin t \cdot \cos t + b^2\sin^2 t\}$$
$$x^2 + y^2 = \{a^2(\sin^2 t + \cos^2 t) + b^2(\cos^2 t + \sin^2 t)\}$$
$$x^2 + y^2 = a^2 + b^2$$

Differentiate w.r.t. x, we get

$2x + 2y\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$ ...(1)

$\therefore \quad y\frac{dy}{dx} = -x$

Again differentiating w.r.t. x, we get

$y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = -1$

Substituting value of $\frac{dy}{dx}$ from (1), we get

$$y\frac{d^2y}{dx^2} + \left(\frac{-x}{y}\right)^2 = -1 \Rightarrow y\frac{d^2y}{dx^2} + \frac{x^2}{y^2} = -1$$
$$\therefore y^3\frac{d^2y}{dx^2} + x^2 + y^2 = 0$$

Question228

The integrating factor of the differential equation $x\frac{dy}{dx} + y\log x = x^2$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $(\log x)^x$

B. $x^{\log x}$

C. $(\log x)^2$

D. $x^{\log(\sqrt{x})}$

Answer: D

Solution:

Given differential equation:

$$x\frac{dy}{dx} + y\log x = x^2$$

Step 1: Make it linear in standard form

Divide the whole equation by x:

$$\frac{dy}{dx} + \frac{y\log x}{x} = x$$

Now, it is in the form:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

where $P(x) = \frac{\log x}{x}$ and $Q(x) = x$.

Step 2: Find the integrating factor (I.F.)

$$I.F. = e^{\int P(x)\,dx}$$

$$I.F. = e^{\int \frac{\log x}{x}\,dx}$$

Step 3: Simplify the integral

Let $\log x = t \Rightarrow \frac{1}{x}dx = dt$

Then:

$$\int \frac{\log x}{x}dx = \int t\,dt = \frac{t^2}{2} = \frac{(\log x)^2}{2}$$

Step 4: Substitute back

$$I.F. = e^{\frac{(\log x)^2}{2}}$$

We can write this as:

$$I.F. = \left(e^{\log x}\right)^{\frac{\log x}{2}} = x^{\frac{\log x}{2}} = x^{\log(\sqrt{x})}$$

✅ **Final Answer:**

$$\boxed{I.F. = x^{\log(\sqrt{x})}}$$

Question229

If the population grows at the rate of 8% per year, then the time taken for thepopulation to be doubled, is (Given $\log 2 = 0 \cdot 6912$) MHT CET 2020 (19 Oct Shift 2)

Options:

A. $6 \cdot 8$ years

B. $10 \cdot 27$ years

C. $8 \cdot 64$ years

D. $4 \cdot 3$ years

Answer: C

Solution:

$\mathrm{P_0}$ be the initial population and let the population after t years be $2\mathrm{P_0}$. then,

$$\frac{\mathrm{dP}}{\mathrm{dt}} = \frac{8\mathrm{P}}{100} \Rightarrow \frac{\mathrm{dP}}{\mathrm{dt}} = \frac{2\mathrm{P}}{25}$$
$$\therefore \frac{\mathrm{dP}}{\mathrm{P}} = \frac{2}{25}\mathrm{dt} \Rightarrow \int \frac{1}{\mathrm{P}}\mathrm{dP} = \frac{2}{25}\int \mathrm{dt}$$

$\log P = \frac{2}{25}t + C$...(1)

At, $t = 0, P = P_0$

$\log P_0 = \frac{2\times 0}{25} + C \Rightarrow C = \log P_0$

$\therefore \log P = \frac{2}{25}t + \log P_0 \Rightarrow \log \frac{P}{P_0} = \frac{2}{25}t$

$\therefore t = \frac{25}{2} \cdot \log\left(\frac{P}{P_0}\right)$

When $P = 2P_0$

$t = \frac{25}{2} \cdot \log\left(\frac{2P_0}{P_0}\right) = \frac{25}{2}\log 2 = 8.64$

Question230

The general solution of the differential equation $(1 + y^2) + \left(x - e^{\tan^{-1} y}\right)\frac{dy}{dx} = 0$ is MHT CET 2020 (19 Oct Shift 2)

Options:

A. $x \cdot e^{\tan^{-1} y} = \frac{\left(e^{\tan^{-1} x}\right)^2}{2} + c$

B. $e^{\tan^{-1} y} = \left(e^{\tan^{-1} x}\right)^2 + c$

C. $x \cdot e^{\tan^{-1} y} = \frac{\left(e^{tan^{-1} y}\right)^2}{2} + c$

D. $e^{\tan^{-1} y} = \left(e^{\tan^{-1} y}\right)^2 + c$

Answer: C

Solution:

$$(1+y^2)+\left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=0$$

$$\therefore \quad \left(x-e^{\tan^{-1}y}\right)\frac{dy}{dx}=-\left(1+y^2\right)\Rightarrow\frac{dy}{dx}=\frac{-(1+y^2)}{x-e^{\tan^{-1}y}}$$

$$\therefore \frac{dx}{dy}=\frac{\left(x-e^{\tan^{-1}y}\right)}{-(1+y^2)}\Rightarrow\frac{dx}{dy}=\frac{-x}{(1+y^2)}+\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\therefore \frac{dx}{dy}+\frac{x}{1+y^2}=\frac{e^{\tan^{-1}y}}{1+y^2}$$

$$\text{I.F.}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

So, the general solution is

$$x\cdot e^{\tan^{-1}y}=\int\frac{e^{\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}dx$$

$$\text{Put } e^{\tan^{-1}y}=t\Rightarrow\frac{e^{\tan^{-1}y}}{1+y^2}dy=dt$$

$$\therefore x\cdot e^{\tan^{-1}y}=\int t\,dt$$

$$\therefore x\cdot e^{\tan^{-1}y}=\frac{t^2}{2}+c\Rightarrow x\cdot e^{\tan^{-1}y}=\frac{\left(e^{\tan^{-1}y}\right)^2}{2}+c$$

Question231

The rate of disintegration of a radio active element at time t is proportional to itsmass at that time. Then the time during which the original mass of $1\cdot 5\mathrm{gm}$. willdisintegrate into its mass of $0.5\mathrm{gm}$. is proportional to MHT CET 2020 (19 Oct Shift 2)

Options:

A. $\log 4$

B. $\log 5$

C. $\log 3$

D. $\log 2$

Answer: C

Solution:

Let m be the mass of the radioactive element at time t.

Then the rate of disintegration is $\frac{dm}{dt}$ which is proportional to m.

$\therefore \frac{dm}{dt} \propto m \Rightarrow \frac{dm}{dt} = -km$, where $k > 0 \therefore \frac{dm}{m} = -kdt \Rightarrow \int \frac{1}{m} dm = -k \int dt + c$

$\therefore \log m = -kt + c$

Initially, i.e. when $t = 0, \ m = 1.5$

$\therefore \log(1.5) = -k \times 0 + c \Rightarrow c = \log\left(\frac{3}{2}\right) \therefore \log m = -kt + \log\left(\frac{3}{2}\right) \Rightarrow \log m - \log \frac{3}{2} = -kt$

$\therefore \log\left(\frac{2m}{3}\right) = -kt$

When $m = 0.5 = \frac{1}{2}$, then

$$\log\left(\frac{2 \times \frac{1}{2}}{3}\right) = -kt \Rightarrow \log\left(\frac{1}{3}\right) = -kt \Rightarrow -\log 3 = -kt$$
$$\therefore t = \frac{1}{k} \log 3$$

$\therefore$ Thus required time is proportional to $\log 3$.

Question232

The differential equation of the circles having their centres on the line $y = 8$ and touching the X -axis is MHT CET 2020 (19 Oct Shift 1)

Options:

A. $(y-8)^2\left[1-\left(\frac{dy}{dx}\right)^2\right] = 64$

B. $(y-8)^2\left[1+\left(\frac{dy}{dx}\right)^2\right] = 64$

C. $(y-8)\left[1+\left(\frac{dy}{dx}\right)^2\right] = 64$

D. $y^2\left(1+\frac{dy}{dx}\right) = 64$

Answer: B

Solution:

Let $(\mathbf{h}, 8)$ be the centre of the circle.

Since circle touches X axis, radius $= 8$

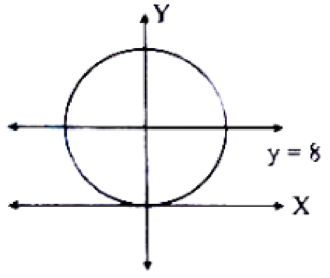
$\therefore (x-h)^2 + (y-8)^2 = (8)^2$...(1)

Differentiating w.r.t. x, we get

$$2(x-h) + 2(y-8)\frac{dy}{dx} = 0$$
$$\therefore \quad (x-h) = -(y-8)\frac{dy}{dx}$$

Substituting value of $(x-h)$ in eq. (1), we get

$$\left[-(y-8)\frac{dy}{dx}\right]^2 + (y-8)^2 = 64$$
$$\therefore (y-8)^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = 64$$



Question233

The integrating factor of differential equation $\left(1+y+x^2y\right)dx + \left(x+x^3\right)dy = 0$ is MHT CET 2020 (19 Oct Shift 1)

Options:

A. $\frac{1}{x}$

B. x

C. $\log x$

D. e^x

Answer: B

Solution:

We have $\left(1+y+x^{2} y\right) d x+\left(x+x^{3}\right) d y=0$

$\therefore \frac{d y}{d x}=\frac{-\left(1+y+x^{2} y\right)}{x\left(1+x^{2}\right)}=\frac{-\left[1+y\left(1+x^{2}\right)\right]}{x\left(1+x^{2}\right)}$

$\therefore \frac{d y}{d x}=\frac{-1}{x\left(1+x^{2}\right)}-\frac{y\left(1+x^{2}\right)}{x\left(1+x^{2}\right)}$

$\therefore \frac{d y}{d x}+\left(\frac{1}{x}\right) y=\frac{-1}{x\left(1+x^{2}\right)}$

$\therefore$ I.F. $=e^{\int \frac{1}{x} d x}=e^{\log x}=x$

Question234

The bacteria increases at the rate proportional to the number of bacteria present. Ifthe original number ' N' doubles in 4 hours then the number of bacteria in 12 hourswill be MHT CET 2020 (19 Oct Shift 1)

Options:

A. 3 N

B. 4 N

C. 6 N

D. 8 N

Answer: D

Solution:

Number of bacteria at the beginning $=N$. The number doubles in every $4 \mathrm{hrs}$.

$\therefore$ Number of bacteria after $4 \mathrm{hrs}=2 \mathrm{~N}$, after $8 \mathrm{hrs}=4 \mathrm{~N}$, after $12 \mathrm{hrs}=8 \mathrm{~N}$

Question235

If the radius of a circular blot of oil is increasing at the rate of $2 \mathrm{~cm} / \mathrm{min}$, then the rateof change of its area when its radius is $3 \mathrm{cms}$ is MHT CET 2020 (19 Oct Shift 1)

Options:

A. $10 \pi \mathrm{cm}^{2} / \mathrm{min}$

B. $12 \pi \mathrm{cm}^{2} / \mathrm{min}$

C. $14 \pi \mathrm{cm}^{2} / \mathrm{min}$

D. $16 \pi \mathrm{cm}^{2} / \mathrm{min}$

Answer: B

Solution:

Given $\frac{dr}{dt} = 2\ \mathrm{cm/min}$ and $\mathrm{r} = 3\mathrm{cms}$

Area $= \pi r^2$

Differentiating w.r.t. t

$\frac{dA}{dt} = \pi \cdot 2r\frac{dr}{dt} = \pi \times 2(3) \times (2) = 12\pi \mathrm{cm}^2/\mathrm{min}$

Question236

A body is heated to $110^\circ\mathrm{C}$ and placed in air at $10^\circ\mathrm{C}$. After 1 hour its temperature is$60^\circ\mathrm{C}$. The additional time required for it to cool to $30^\circ\mathrm{C}$ is MHT CET 2020 (19 Oct Shift 1)

Options:

A. $\left(\frac{\log 2}{\log 5} + 1\right)$ hours

B. $\left(\frac{\log 5}{\log 2}\right)$ hours

C. $\left(\frac{\log 5}{\log 2} - 1\right)$ hours

D. $\left(\frac{\log 2}{\log 5}\right)$ hours

Answer: C

Solution:

Let θ be the temperature of body at time t. Temperature of air is given to be $10^{\circ}\mathrm{C}. = \theta_0$ (say).

$$\frac{\mathrm{d}\theta}{\mathrm{dt}} \propto (\theta - \theta_0)$$
$$\frac{\mathrm{d}\theta}{\mathrm{dt}} = -\mathrm{k}(\theta - \theta_0), \mathrm{k} > 0$$
$$\therefore \int \frac{\mathrm{d}\theta}{\theta - \theta_0} = \int -\mathrm{kt} \quad \Rightarrow \log|\theta - \theta_0| = -\mathrm{kt} + \log \mathrm{c} \ldots (1)$$

$\therefore \left(\frac{\theta-\theta_0}{\mathrm{c}}\right) = \mathrm{e}^{-\mathrm{kt}} \therefore \theta = \theta_0 + \mathrm{ce}^{-\mathrm{kt}}$ When $\mathrm{t} = 0, \theta = 110$ and $\theta_0 = 10$, $\quad \therefore \theta = 10 + 100\mathrm{e}^{-\mathrm{kt}}$

$110 = 10 + \mathrm{c} \Rightarrow \mathrm{c} = 100 \quad \therefore \quad \mathrm{c}$ When $\theta = 60^{\circ}\mathrm{C}, \mathrm{t} = 1$ Substituting value in equation (1),

$$\therefore \quad \log\left(\frac{60-10}{100}\right) = -\mathrm{k} \times 1$$

$$\mathrm{k} = -\log\left(\frac{1}{2}\right)$$

$$\therefore \quad \log\left(\frac{\theta-10}{100}\right) = \mathrm{t} \cdot \log\left(\frac{1}{2}\right)$$

When $\theta = 30^{\circ}\mathrm{C}$, then

$$\log\left(\frac{30-10}{100}\right) = \mathrm{t} \cdot \log\left(\frac{1}{2}\right)$$

$$\log\left(\frac{20}{100}\right) = t \cdot \log \frac{1}{2}$$

$$-\log 5 = -t \log 2 \Rightarrow t = \frac{\log 5}{\log 2}$$

$\therefore$ Additional time required is $\frac{\log 5}{\log 2} - 1$

Question237

The order and degree of the differential equation $\sqrt{1 + \frac{1}{\left(\frac{dy}{dx}\right)^2}} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}}$ are respectively

MHT CET 2020 (19 Oct Shift 1)

Options:

A. 3,2

B. 2,3

C. 2,2

D. 3,3

Answer: B

Solution:

By raising the given equation to the power of 2 , we get

$$1+\frac{1}{\left(\frac{dy}{dx}\right)^2}=\left(\frac{d^2y}{dx^2}\right)^3$$

$$\therefore\left(\frac{dy}{dx}\right)^2+1=\left(\frac{d^2y}{dx^2}\right)^3\left(\frac{dy}{dx}\right)^2$$

It's order is 2 and degree is 3 .

Question238

The integrating factor of the differential equation siny $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)=\cos y(1-x\cos y)$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. e^{-x}

B. $e^{-\cos y}$

C. e^{-y}

D. $e^{\sin y}$

Answer: A

Solution:

$\sin y\frac{dy}{dx}=\cos y(1-x\cos y)\Rightarrow\sin y\frac{dy}{dx}=\cos y-x\cos^2 y$

Dividing both sides by $\cos^2 y$, we get

$\sec y\tan y\frac{dy}{dx}=\sec y-x$

$\sec y\cdot\tan y\frac{dy}{dx}-\sec y=-x$

Put $\sec y=v\Rightarrow\sec y\tan y dy=dx$

$\therefore\frac{\mathrm{dv}}{\mathrm{dx}}-\mathrm{v}=-\mathrm{x}$...(1)

$\therefore$ I.F. $=\mathrm{e}^{-\int 1\mathrm{dx}}=\mathrm{e}^{-\mathrm{x}}$

Question239

The micro-organisms double themselves in 3 hours. Assuming that the quantity increases at a rate proportional to it self, then the number of times it multiplies themselves in 18 years is MHT CET 2020 (16 Oct Shift 2)

Options:

A. 32

B. 64

C. 128

D. 40

Answer: B

Solution:

Let initial number of microorganisms be N. The microorganisms double themselves in 3 hours. ∴ Number of microorganisms after

3 hours $= 2 \times \mathrm{N} = 2\,\mathrm{N}$

o hours $= 2 \times 2\,\mathrm{N} = 4\,\mathrm{N}$

9 hours $= 2 \times 4\,\mathrm{N} = 8\,\mathrm{N}$

12 hours $= 2 \times 8\,\mathrm{N} = 16\,\mathrm{N}$

15 hours $= 2 \times 16\,\mathrm{N} = 32\,\mathrm{N}$

18 hours $= 2 \times 32\,\mathrm{N} = 64\,\mathrm{N}$

Question240

The rate at which the metal cools in moving air is proportional to the difference of tempratures between the metal and air. If the air temperature is $290\ \mathrm{K}$ and the metal temperature drops from $370\ \mathrm{K}$ to $330\ \mathrm{K}$ in 10 minutes, then the time required to drop the temperature upto $295\ \mathrm{K}$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. 40min

B. 20min

C. 35min

D. 30min

Answer: A

Solution:

We know that $\frac{dT}{dt} = -k(T - Tm)$, where

k is proportionality constant, T = Temperature of body, Tm = Temperature of surrounding.

$\therefore \frac{dT}{dt} = -k(T - 290) \Rightarrow \int \frac{dT}{T-290} = \int -kdt$

$\log|T - 290| = -kt + C$

Initially, $t = 0,\ T = 370$

$\therefore \log|370 - 290| = 0 + C \Rightarrow C = \log|80|$

$\therefore \log|T - 290| = -kt + \log|80|$

When $t = 10,\ T = 330$

$\therefore \log|330 - 290| = -10k + \log|80|$

$\therefore \log\left[\left[\frac{40}{80}\right]\right] = -10k \Rightarrow k = \frac{1}{10}\log 2$

$\therefore \log[|T - 290|] = -\frac{\log 2}{10}t + \log|80|$

When $T = 295$, we write

$\log[295 - 290]] = -\frac{\log 2}{10}t + \log 80$

$\therefore \frac{(\log 5 - \log 80)}{\log 2}\backslash(-10) = 1$

$\frac{\log\left(\frac{1}{16}\right)}{\log 2}\backslash(-10) = 1$

$\therefore t = \frac{-4\log 2}{\log 2}\backslash(-10)$

$= 40$

Question241

The particular solution of the differential equation $y\left(\frac{dx}{dy}\right) = x\log x$ at $x = e$ and $y = 1$ is MHT CET 2020 (16 Oct Shift 2)

Options:

A. $e^{xy} = 2$

B. $x = e^y$

C. $xy = 2$

D. $\log x = 2y$

Answer: B

Solution:

$y\left(\frac{dx}{dy}\right) = x \cdot \log x$

$\therefore \int \frac{1}{x \cdot \log x} dx = \int \frac{1}{y} dy$

$\therefore \log|\log x| = \log y + \log c$

We have $x = e$ and $y = 1$

$\therefore \log|\log e| = \log 1 + \log c \Rightarrow \log c = 0$

$\therefore \log|\log x| = \log y \Rightarrow \log x = y \Rightarrow x = e^y$

Question242

The order and degree of the differential equation $\left[1+\left[\frac{dy}{dx}\right]^3\right]^{\frac{7}{3}} = 7\frac{d^2y}{dx^2}$ are respectively.

MHT CET 2020 (16 Oct Shift 2)

Options:

A. 2,1

B. 2,3

C. 1,2

D. 3,2

Answer: B

Solution:

Given equation:

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}}=7\frac{d^2y}{dx^2}$$

Step 1: Identify the highest order derivative

The highest order derivative in the equation is

$$\frac{d^2y}{dx^2}$$

So,

$$\text{Order}=2$$

Step 2: Remove fractional powers (to find degree)

To find the **degree**, the equation must be free from fractions or radicals involving derivatives.

Raise both sides to power 3 to remove the fractional exponent $\frac{7}{3}$:

$$\left(\left[1+\left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}}\right)^3=(7\frac{d^2y}{dx^2})^3$$

Simplify:

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^7=343\left(\frac{d^2y}{dx^2}\right)^3$$

Now, the **highest derivative** $\frac{d^2y}{dx^2}$ has power **3**.

✅ **Order = 2, Degree = 3**

Question243

The differential equation obtained from the function $y=a(x-a)^2$ is MHT CET 2020 (16 Oct Shift 1)

Options:

A. $8y^2=\left(\frac{dy}{dx}\right)^2\left[x-\frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2$

B. $8y^2=\left(\frac{dy}{dx}\right)^2\left[x+\frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2$

C. $2y^2=\left(\frac{dy}{dx}\right)^2\left[x-\frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2$

D. $4y^2=\left(\frac{dy}{dx}\right)^2\left[x-\frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2$

Answer: D

Solution:

Given:

$$y = a(x-a)^2$$

We need to **eliminate the constant** a to form a differential equation.

Step 1: Differentiate both sides

$$\frac{dy}{dx} = a \cdot 2(x-a)$$

$$\Rightarrow \frac{dy}{dx} = 2a(x-a)$$

Step 2: Express a in terms of $x, y, \frac{dy}{dx}$

From the original equation:

$$y = a(x-a)^2$$

Let's first express $(x-a)$ from Step 1:

$$x - a = \frac{1}{2a}\frac{dy}{dx}$$

Substitute this in $y = a(x-a)^2$:

$$y = a\left(\frac{1}{2a}\frac{dy}{dx}\right)^2 = \frac{1}{4a}\left(\frac{dy}{dx}\right)^2$$

$$\Rightarrow a = \frac{1}{4y}\left(\frac{dy}{dx}\right)^2$$

Step 3: Substitute a back into $x - a = \frac{1}{2a}\frac{dy}{dx}$

$$x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2 = \frac{1}{2a}\frac{dy}{dx}$$

But $a = \frac{1}{4y}\left(\frac{dy}{dx}\right)^2$,
so $\frac{1}{a} = \frac{4y}{\left(\frac{dy}{dx}\right)^2}$.

Substitute:

$$x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2 = \frac{1}{2} \cdot \frac{4y}{\left(\frac{dy}{dx}\right)^2} \cdot \frac{dy}{dx}$$

Simplify:

$$x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2 = \frac{2y}{\frac{dy}{dx}}$$

Step 4: Rearrange and simplify

Multiply both sides by $\frac{dy}{dx}$:

$$\left(\frac{dy}{dx}\right)\left[x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right] = 2y$$

Square both sides to remove root and get a standard form:

$$\left(\frac{dy}{dx}\right)^2\left[x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2 = 4y^2$$

✅ **Final Answer:**

$$\boxed{4y^2 = \left(\frac{dy}{dx}\right)^2\left[x - \frac{1}{4y}\left(\frac{dy}{dx}\right)^2\right]^2}$$

Question244

The bacteria increases at the rate proportional to the number of bacteria present. If the original number 'N' doubles in 4 hours, then the number of bacteria in 12 hours will be MHT CET 2020 (16 Oct Shift 1)

Options:

A. 4N

B. 3N

C. 8N

D. 6N

Answer: C

Solution:

Let x be the number of bacteria at time t.

$\frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = kx \Rightarrow \int \frac{dx}{x} = \int k \cdot dt$

$\log x = kt + c$

Initially, i.e. when $t = 0$, let $x = x_0$

$$\begin{aligned} \log x_0 &= k \times 0 + c \\ \log x &= kt + \log x_0 \end{aligned} \quad \Rightarrow c = \log x_0$$

$$\log x - \log x_0 = kt \quad \Rightarrow \log\left(\frac{x}{x_0}\right) = kt \ldots(1)$$

Since the number doubles in 4hrs, i.e.

$$t = 4 \text{ and } x = 2x_0$$
$$\therefore \log\left(\frac{2x_0}{x_0}\right) = 4k \quad \Rightarrow k = \frac{1}{4}\log 2$$

$$\text{When } t = 12, \log\left(\frac{x}{x_0}\right) = \frac{12}{4}\log 2 = 3\log 2$$
$$\therefore \log\left(\frac{x}{x_0}\right) = \log 8 \quad \Rightarrow x = 8x_0$$

This problem can be alternatively solved as follows :

Original number $= \mathrm{N}$.

After four hours, number $= 2\ \mathrm{N}$

After eight hours, number $= 4\ \mathrm{N}$

After twelve hours, number $= 8\ \mathrm{N}$

Question245

The differential equation of all lines perpendicular to the line $5x + 2y + 7 = 0$ is MHT CET 2020 (16 Oct Shift 1)

Options:

A. 3dy-2dx=0

B. 2dy-5dx=0

C. 2dy-3dx=0

D. 5dy-2dx=0

Answer: D

Solution:

Given line $5x + 2y + 7 = 0$ has slope $\frac{-5}{2}$. Hence eq. of required line is

$$y = \frac{2}{5}x + c \Rightarrow 2x - 5y + c = 0$$
$$\therefore 2 - 5\frac{dy}{dx} = 0 \Rightarrow 2 = 5\frac{dy}{dx} \Rightarrow 2dx = 5dy \Rightarrow 5dy - 2dx = 0$$

Question246

The rate of decay of certain substance is directly proportional to the amount present at that instant. Initially, there are 27 gms of certain substance and 3 hours later it is found that 8gms are left, then the amount left after one more hour is MHT CET 2020 (16 Oct Shift 1)

Options:

A. $\frac{19}{3}$gms

B. $\frac{20}{3}$gmS

C. $\frac{17}{3}$gms

D. $\frac{16}{3}$gms

Answer: D

Solution:

Let x gms be the amount of the substance left at time t. Then the rate of decay be $\frac{dx}{dt}$, which is proportional to x.

$$\therefore \frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = -k \cdot x, \text{ where } k > 0$$
$$\therefore \int \frac{1}{x} dx = \int -k \cdot dt$$
$$\therefore \log x = -kt + c$$
Initially i.e. when $t = 0, x = 27$.
$$\log 27 = -k \times 0 + C \Rightarrow C = \log 27$$
$$\therefore \log x = -kt + \log 27$$
$$\therefore \log x - \log 27 = -kt$$
Now, when $t = 3, x = 8$

$$\therefore \quad \log\left(\frac{8}{27}\right) = -3k \Rightarrow -3k = \log\left(\frac{2}{3}\right)^3 = 3\log\left(\frac{2}{3}\right)$$
$$k = -\log\left(\frac{2}{3}\right)$$
$$\therefore \quad \log\left(\frac{x}{27}\right) = t \cdot \log\left(\frac{2}{3}\right)$$
When $t = 4$,
$$\log\left(\frac{x}{27}\right) = 4\log\left(\frac{2}{3}\right) = \log\left(\frac{2}{3}\right)^4$$
$$\frac{x}{27} = \frac{16}{81} \Rightarrow x = \frac{16}{3}\text{gms}$$

Question247

The intearrati the differential equation $\left(1+x^2\right) dt=\left(\tan^{-1} x-t\right) dx$ MHT CET 2020 (16 Oct Shift 1)

Options:

A. $-e^{\frac{\left(\tan^{-1} x\right)^2}{2}}$

B. $-e^{\tan^{-1} x}$

C. $e^{\frac{\left(\tan^{-1} x\right)^2}{2}}$

D. $e^{\tan^{-1} x}$

Answer: D

Solution:

$$\frac{dt}{dx}=\frac{\tan^{-1}-t}{1+x^2}$$
$$\therefore \frac{dt}{dx}+\frac{t}{1+x^2}=\frac{\tan^{-1} x}{1+x^2}$$
$$\text{I.F.}=e^{\int \frac{1}{1+x^2} dx}=e^{\tan^{-1} x}$$

Question248

Radium decomposes at a rate proportional to the amount present. If half the orignal amount disappears in 1600 yrs, then the percentage loss in 100 years is (Given $\log 2=0.6912 \& e^{-0.04320}=0 \cdot 9576$) MHT CET 2020 (15 Oct Shift 2)

Options:

A. $3 \cdot 24\%$

B. $5 \cdot 24\%$

C. $2 \cdot 24\%$

D. $4 \cdot 24\%$

Answer: D

Solution:

Let $R =$ Amount of radium present at time t.

We have $\frac{dR}{dt} \propto R \Rightarrow \frac{dR}{t} = kR \quad \rightarrow \int \frac{dR}{k} = \int kdt$

$\therefore \log R = kt + C$...(1)

when $t = 0$, let $R = R_0$ so we get

$\log R_0 = 0 + c \Rightarrow c = \log R$

$\log \frac{R}{R_0} = kt$

$(\therefore)$

When $t = 1600\text{yrs}, \mathrm{R} = \frac{1}{2}\mathrm{R}_0$

$\therefore \log \frac{1}{2} \frac{R_0}{R_0} = 1000 \Rightarrow \log \frac{1}{2} - 1000k \therefore k = \frac{1}{1000} \log \frac{1}{2} - \frac{1}{1000}(0 - kg_2) = \frac{-00012}{1000}$

$\therefore k = -0.000432t$

When $\mathrm{t} = 100$, we get

$\therefore \log \frac{R}{R_0} = -0.0432 \Rightarrow \frac{R}{R_0} = e^{-4043}$

$\therefore \frac{R}{R_0} = 0.9576 \Rightarrow R = 0.9576R_0$

$\% \text{ loss} = \frac{R_0 - 0.9576R_0}{R_0} \times 100\%$

$= 0.0424 \times 100\% = 4.24\%$

Question249

The solution of the differential equation $\log\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) = 9x - 6\mathrm{y} + 6$ is(given that $\mathrm{y} = 1$ when $x = 0$) MHT CET 2020 (15 Oct Shift 2)

Options:

A. $3e^{6y} = 2e^{9x-6} + e^{6}$

B. $3e^{6y} = 2e^{9x+6} + e^{6}$

C. $3e^{6y} = 2e^{9x+6} - e^{6}$

D. $3e^{6y} = 2e^{9x-6} - e^{6}$

Answer: B

Solution:

$\log\left(\frac{dy}{dx}\right) = 9x - 6y + 6 \Rightarrow \frac{dy}{dx} = e^{9x-6y+6}$

$\therefore \frac{dy}{dx} = e^{9x+6} \cdot e^{-6y} \Rightarrow \frac{dy}{e^{-6y}} = e^{9x} \cdot e^{6} dx$

$\therefore \int e^{6y} dy = e^{6} \int e^{9x} dx$

$\frac{e^{6y}}{6} = \frac{e^{6} e^{9x}}{9} + C$

We have $x = 0, y = 1$

$\therefore \frac{e^6}{6} = \frac{e^6}{9} + C \Rightarrow C = \frac{e^6}{18}$

$\therefore \frac{e^{6y}}{6} = \frac{e^{6} e^{9x}}{9} + \frac{e^6}{18} \Rightarrow 3e^{6y} = 2e^{6+9x} + e^6$

Question250

The differential equation of the family of curves $\mathrm{y} = e^{x}(\mathrm{~A} \cos x + \mathrm{B} \sin x)$, where$\mathrm{A}$ and B are arbitrary constants is MHT CET 2020 (15 Oct Shift 2)

Options:

A. $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2} + 2\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) + 2\mathrm{y} = 0$

B. $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2} - 2\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) - 2\mathrm{y} = 0$

C. $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2} + 2\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) - 2\mathrm{y} = 0$

D. $\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2} - 2\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) + 2\mathrm{y} = 0$

Answer: D

Solution:

Given:

$$y = e^x(A\cos x + B\sin x)$$

We need to **eliminate A and B** to form a differential equation.

Step 1: Differentiate once

$$\frac{dy}{dx} = e^x(A\cos x + B\sin x) + e^x(-A\sin x + B\cos x)$$

Simplify:

$$\frac{dy}{dx} = e^x[(A+B)\cos x + (B-A)\sin x]$$

Step 2: Differentiate again

$$\frac{d^2y}{dx^2} = e^x[(A+B)\cos x + (B-A)\sin x] + e^x[-(A+B)\sin x + (B-A)\cos x]$$

Simplify:

$$\frac{d^2y}{dx^2} = e^x[2B\cos x - 2A\sin x]$$

Step 3: Substitute and eliminate A, B

From the given expressions, when simplified, we get:

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0$$

✅ **Final Answer:**

$$\boxed{\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0}$$

Question251

The growth of population is proportional to the number present. If the population of a colony doubles is 50 years, then the population will become triple in _____ years MHT CET 2020 (15 Oct Shift 2)

Options:

A. $5\left(\frac{\ell og 2}{\ell og 3}\right)$ yrs

B. $50\left(\frac{\log 3}{\ell og 2}\right)$ yrs

C. $5\left(\frac{\log 3}{\log 2}\right)$ yrs

D. $50\left(\frac{\log 2}{\ell og 3}\right)$ yrs

Answer: B

Solution:

Let $P_0 =$ Initial population

Given $\frac{dP}{dt} \propto P \Rightarrow \frac{dP}{dt} = \lambda P \Rightarrow \int \frac{dP}{P} = \int \lambda dt$

$\log |P| = \lambda t + C \ldots (1)$

At $t = 0, \quad P = P_0$ we get $\log P_0 = 0 + C \Rightarrow C = \log P_0$

$\log P = \lambda t + \log P_0 \Rightarrow \log\left(\frac{P}{P_0}\right) = \lambda t$

When $P = 2P_0, t = 50 \Rightarrow \log\left(\frac{2P_0}{P_0}\right) = 50\lambda$

$\therefore \log 2 = 50\lambda \Rightarrow \lambda = \frac{1}{50}\log 2$

$\therefore \log\left(\frac{P}{P_0}\right) = \frac{t}{50}(\log 2)$

When $P = 3P_0$ we get

$\log 3 = \frac{t}{50}(\log 2) \Rightarrow t = 50\left(\frac{\log 3}{\log 2}\right)$ yrs.

Question252

The population $P(t)$ of a certain mouse species at time t satisfies the differential equation $\frac{dP(t)}{dt} = 0 \cdot 5P(t) - 450\cdot$ If $P(0) = 850$, then the time at which the population becomes zero is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $\left(\frac{1}{2}\right) \log 18$

B. $\log 18$

C. $2 \log 18$

D. $\log 9$

Answer: C

Solution:

$\frac{dp(t)}{dt} = 0.5p(t) - 450$

$\frac{dp(t)}{dt} = \frac{1}{2}p(t) - 450 \Rightarrow \frac{dp(t)}{dt} = \frac{p(t)-900}{2}$

$\therefore 2\int \frac{d[p(t)]}{p(t)-900} = \int dt \Rightarrow 2\log|p(t) - 900| = t + c$

When $t = 0, p(t) = 850$

$\therefore 2\log|850 - 900| = 0 + c \Rightarrow c = 2\log 50$

$\therefore 2\log|p(t) - 900| = t + 2\log 50$

When $p(t) = 0$, we write

$2\log|0 - 900| = t + 2\log 50$

$\therefore t = 2\log\left|\frac{900}{50}\right|$

$t \quad = 2\log 18$

Question253

The differential equation whose solution is $y = c_1 \cos ax + c_2 \sin ax$(Where c_1 and c_2 are arbitrary constants) is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $\frac{d^2y}{dx^2} - a^2y = 0$

B. $\frac{d^2y}{dx^2} + a^2y = 0$

C. $\frac{d^2y}{dx^2} + ay^2 = 0$

D. $\frac{d^2y}{dx^2} + y^2 = 0$

Answer: B

Solution:

Given:

$$y = c_1 \cos(ax) + c_2 \sin(ax)$$

where c_1 and c_2 are arbitrary constants.
We need to eliminate c_1 and c_2.

Step 1: Differentiate once

$$\frac{dy}{dx} = -ac_1 \sin(ax) + ac_2 \cos(ax)$$

Step 2: Differentiate again

$$\frac{d^2y}{dx^2} = -a^2c_1 \cos(ax) - a^2c_2 \sin(ax)$$

Step 3: Simplify
Factor out $-a^2$:

$$\frac{d^2y}{dx^2} = -a^2(c_1 \cos(ax) + c_2 \sin(ax))$$

But $(c_1 \cos(ax) + c_2 \sin(ax)) = y$

So,

$$\frac{d^2y}{dx^2} = -a^2y$$

or,

$$\boxed{\frac{d^2y}{dx^2} + a^2y = 0}$$

Question254

The rate of decay of mass of a certain substance at time 't' is proportional to themass at that instant. The time during which the original mass of m_0 gm. will be left to $m_1$gm. is (K is constant of proportionality) MHT CET 2020 (15 Oct Shift 1)

Options:

A. $\mathrm{K} \log\left(\frac{m_1}{m_0}\right)$

B. $\left(\frac{1}{\mathrm{K}}\right) \log\left(\frac{m_1}{m_0}\right)$

C. $\left(\frac{1}{\mathrm{K}}\right) \log\left(\frac{m_0}{m_1}\right)$

D. $\mathrm{K} \log\left(\frac{m_0}{m_1}\right)$

Answer: C

Solution:

Given $\frac{\mathrm{dm}}{\mathrm{dt}} \propto \mathrm{m} \Rightarrow \frac{\mathrm{dm}}{\mathrm{dt}} = -\mathrm{Km} \Rightarrow \int \frac{\mathrm{dm}}{\mathrm{m}} = \int -\mathrm{Kdt}$

$\therefore \log \mathrm{m} = -\mathrm{Kt} + \mathrm{c}$

When $\mathrm{t} = 0,\ \mathrm{m} = \mathrm{m}_0$

$\therefore \log \mathrm{m}_0 = 0 + \mathrm{c} \Rightarrow \mathrm{c} = \log \mathrm{m}_0$

$$\log \mathrm{m} = -\mathrm{Kt} + \log \mathrm{m}_0$$

$\therefore \log\left(\frac{\mathrm{m}}{\mathrm{m}_0}\right) = -\mathrm{Kt}$

When $\mathrm{m} = \mathrm{m}_1$, we get

$$\log\left(\frac{\mathrm{m}_1}{\mathrm{m}_0}\right) = -\mathrm{Kt}$$

$\mathrm{t} = \frac{-1}{\mathrm{K}} \log\left(\frac{\mathrm{m}_1}{\mathrm{m}_0}\right) = \frac{1}{\mathrm{K}} \log\left(\frac{\mathrm{m}_0}{\mathrm{m}_1}\right)$

Question255

The particular solution of the differential equation $\cos\left(\frac{\mathrm{dy}}{\mathrm{d}x}\right) = a$, under the conditions $\mathrm{a} \epsilon \mathrm{R}$ and $y(0) = 2$ is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $\cos\left(\frac{x-2}{y-2}\right) = a$

B. $\cos^{-1}\left(\frac{y-2}{x}\right) = a$

C. $\cos\left(\frac{y-2}{x}\right) = a$

D. $\cos\left(\frac{x-2}{y+2}\right) = a$

Answer: C

Solution:

$$\cos\left(\frac{dy}{dx}\right)=a\Rightarrow\frac{dy}{dx}=\cos^{-1}a$$

$$\int dy=\int\cos^{-1}a dx$$

$$\therefore y=x\cos^{-1}a+c\ldots(1)$$

We have $x=0, y=2$

$$\therefore\quad 2=0+c\Rightarrow c=2$$

$$\therefore\quad y=x\cos^{-1}a+2$$

$$\therefore y-2=x\cos^{-1}a\Rightarrow\frac{y-2}{x}=\cos^{-1}a\Rightarrow\cos\left(\frac{y-2}{x}\right)=a$$

Question256

The general solution of the differential equation $\left(1-x^2\right)\frac{dy}{dx}+2xy=x\left(1-x^2\right)^{\frac{1}{2}}$ is MHT CET 2020 (15 Oct Shift 1)

Options:

A. $y=\sqrt{1-x^2}+c\left(1-x^2\right)$

B. $y=2\sqrt{1-x^2}+c$

C. $y=2\sqrt{1-x^2}+c\left(1+x^2\right)$

D. $y\sqrt{1-x^2}=c\left(1-x^2\right)$

Answer: A

Solution:

$$\left(1-x^2\right)\frac{dy}{dx}+2xy=x\left(1-x^2\right)^{\frac{1}{2}}$$

$$\therefore\frac{dy}{dx}+\frac{2xy}{1-x^2}=\frac{x}{\left(1-x^2\right)^{\frac{1}{2}}}$$

$$\text{I.F.}=e^{\int\frac{2x}{1-x^2}dx}=e^{-\int\frac{-2x}{1-x^2}dx}=e^{-\log\left(1-x^2\right)}=e^{\log\left(\frac{1}{1-x^2}\right)}=\frac{1}{1-x^2}$$

$$\therefore\left(\frac{1}{1-x^2}\right)=\int\frac{x}{\left(1-x^2\right)^{\frac{1}{2}}}\times\frac{1}{\left(1-x^2\right)}dx$$

$$=\frac{-1}{2}\int\frac{-2x}{\left(1-x^2\right)^{3/2}}dx=\left(-\frac{1}{2}\right)\frac{\left(1-x^2\right)^{-\frac{1}{2}}}{\left(-\frac{1}{2}\right)}+c$$

$$y\left(\frac{1}{1-x^2}\right)=\frac{1}{\sqrt{1-x^2}+c\Rightarrow y}=\sqrt{1-x^2}+c\left(1-x^2\right)$$

Question257

If the population grows at the rate 5% peryear, then the time taken for the populationto become double is (Given $\log 2=0.6912$) MHT CET 2020 (14 Oct Shift 2)

Options:

A. 13.8275 years

B. 13.624 years

C. 13.725 years

D. 13.8240 years

Answer: D

Solution:

Let initial population $= P_0$ and let P be the population at time t We have $\frac{\mathrm{dP}}{\mathrm{dt}} = \frac{5\mathrm{P}}{100}$

$\therefore \frac{dP}{dt} = \frac{P}{20} \Rightarrow \int \frac{dP}{P} = \int \frac{1}{20} dt$

$\therefore \log P = \frac{1}{20} t + C$....(1)

When $t = 0, P = P_0$

$\therefore \log \mathrm{P}_0 = 0 + \mathrm{C} \Rightarrow \mathrm{C} = \log \mathrm{P}_0$

From (1), $\log\left(\frac{P}{P_0}\right) = \frac{t}{20}$....(2)

When $P = 2P_0$, we get

$\log 2 = \frac{t}{20} \Rightarrow t = 20(\log 2) = 20 \times 0.6912 = 13.8240$ years

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Question258

$y = mx + \frac{2}{m}$ is the general solution of MHT CET 2020 (14 Oct Shift 2)

Options:

A. $y\left(\frac{dy}{dx}\right)^2 = x\left(\frac{dy}{dx}\right) + 2$

B. $y = x\frac{dy}{dx} + 2$

C. $y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + 2$

D. $y\left(\frac{dy}{dx}\right) = x + 2$

Answer: C

Solution:

$$\text{Given } \mathrm{y} = \mathrm{mx} + \frac{2}{\mathrm{m}} \ldots (1)$$
$$\therefore \frac{\mathrm{dy}}{\mathrm{dx}} = (\mathrm{m} \times 1) + 0 \Rightarrow \mathrm{m} = \frac{\mathrm{dy}}{\mathrm{dx}}$$

Putting value of m in equation (1), we get

$$y = x\frac{dy}{dx} + \frac{2}{\left(\frac{dy}{dx}\right)} \Rightarrow y\frac{dy}{dx} = x\left(\frac{dy}{dx}\right)^2 + 2$$

Question259

The rate of decay of mass of certain substance at time tis proportional to thett that instant. The time during which the original mass of m_0 gram will be lefthtm, gram is $(k$ is constant of proportionality MHT CET 2020 (14 Oct Shift 2)

Options:

A. $\frac{1}{k}\log\left(\frac{m_1}{m_0}\right)$

B. $k\log\left(\frac{m_0}{m_1}\right)$

C. $k\log\left(\frac{m_1}{m_0}\right)$

D. $\frac{1}{k}\log\left(\frac{m_0}{m_1}\right)$

Answer: D

Solution:

Let m be the initial mass of substance at time t.

$$\therefore \frac{\mathrm{dm}}{\mathrm{dt}} \propto \mathrm{km} \Rightarrow \frac{\mathrm{dm}}{\mathrm{dt}} = -\mathrm{km} \quad \Rightarrow \int \frac{\mathrm{dm}}{\mathrm{m}} = \int -\mathrm{kdt}$$

$$\therefore \log \mathrm{m} = -\mathrm{kt} + \mathrm{c} \ldots (1)$$

$$\text{When } \mathrm{t} = 0,\ \mathrm{m} = \mathrm{m_0} \text{ we get}$$

$$\log \mathrm{m_0} = 0 + \mathrm{c} \Rightarrow \mathrm{c} = \log \mathrm{m_0}$$

$$\text{From } (1), \log\left(\frac{\mathrm{m}}{\mathrm{m_0}}\right) = -\mathrm{kt} \Rightarrow \mathrm{t} = \frac{-1}{\mathrm{k}}\log\left(\frac{\mathrm{m}}{\mathrm{m_0}}\right)$$

When $m = m_1$, we get

$$t = \frac{-1}{k}\log\left(\frac{m_1}{m_0}\right) \Rightarrow t = \frac{1}{k}\log\left(\frac{m_0}{m_1}\right)$$

Question260

$$If \frac{x}{x-y} = \log\left(\frac{a}{x-y}\right), \text{ then } \frac{dy}{dx} =$$

MHT CET 2020 (14 Oct Shift 2)

Options:

A. $2 + \frac{1}{y}$

B. $\frac{2y-x}{y}$

C. $\frac{2x-y}{x}$

D. $\frac{x-2y}{y}$

Answer: B

Solution:

$\frac{x}{x-y} = \log a - \log(x-y)$

$\therefore \log(x-y) + \frac{x}{x-y} = \log a$

$\therefore \frac{1}{(x-y)}\left(1 - \frac{dy}{dx}\right) + \left[\frac{(x-y)(1)-(x)\left(1-\frac{dy}{dx}\right)}{x-y}\right] = 0$

$\therefore \left[\frac{1}{x-y} - \frac{1}{x-y}\frac{dy}{dx}\right] + \left[\frac{x-y-x+x\frac{dy}{dx}}{x-y}\right] = 0$

$\therefore \frac{1}{x-y} - \frac{1}{x-y}\frac{dy}{dx} - \frac{y}{(x-y)^2} + \frac{x}{(x-y)^2} = 0$

$\therefore \quad (x-y) - (x-y)\frac{dy}{dx} - y + x\frac{dy}{dx} = 0$

$\therefore \frac{dy}{dx} = \frac{x-2y}{-y} = \frac{2y-x}{y}$

Question261

The particular solution of the differential equation $\left(y + x \cdot \frac{dy}{dx}\right) \cdot \sin xy = \cos x$at $x = 0$ is MHT CET 2020 (14 Oct Shift 2)

Options:

A. $\sin x + \cos xy = 1$

B. $\cos x - \sin xy = 1$

C. $\sin x - \cos xy = 1$

D. $\cos x + \sin xy = 1$

Answer: A

Solution:

We have $\left(y+x\frac{dy}{dx}\right)\sin xy=\cos x$

Put $xy=u\Rightarrow x\frac{dy}{dx}+y=\frac{du}{dx}$

$\therefore\left(\frac{du}{dx}\right)\sin u=\cos x$

$\therefore\int\sin u du=\int\cos x dx\Rightarrow-\cos u=\sin x+c\Rightarrow-\cos xy=\sin x+c$

When $x=0$, we get

$-\cos 0=0+c\Rightarrow c=-1$

$\therefore-\cos xy=\sin x-1\Rightarrow\sin x+\cos xy=1$

Question262

Solution of the differential equation $\frac{dy}{dx}+2y=e^{-x}$ is MHT CET 2020 (14 Oct Shift 2)

Options:

A. $ye^{x}=x+c$

B. $ye^{2x}=x+c$

C. $ye^{x}=e^{2x}+c$

D. $ye^{2x}=e^{x}+c$

Answer: D

Solution:

$\frac{dy}{dx}+2y=e^{-x}$

I.F. $=e^{2\int dx}=e^{2x}$

$\therefore ye^{2x}=\int e^{2x}\cdot e^{-x}dx+c$

$\therefore ye^{2x}=\int e^{x}dx+c\Rightarrow ye^{2x}=e^{x}+c$

Question263

If the population grows at the rate of 8% per year, then the time taken for thepopulation to be doubled is (given $\log 2=0\cdot 6912$) MHT CET 2020 (14 Oct Shift 1)

Options:

A. 8.64 years

B. 6.8 years

C. 10.27 years

D. $4 \cdot 3$ years

Answer: A

Solution:

Let P_0 be the initial population and let the population after t years be $2P_0$. then,

$$\frac{dP}{dt} = \frac{8P}{100} \Rightarrow \frac{dP}{dt} = \frac{2P}{25}$$
$$\therefore \frac{dP}{P} = \frac{2}{25}dt \Rightarrow \int \frac{1}{P}dP = \frac{2}{25}\int dt$$
$$\log P = \frac{2}{25}t + C \ldots (1)$$

$$\text{At, } t = 0, P = P_0$$
$$\log P_0 = \frac{2\times 0}{25} + C \Rightarrow C = \log P_0$$
$$\therefore \log P = \frac{2}{25}t + \log P_0 \Rightarrow \log\frac{P}{P_0} = \frac{2}{25}t$$
$$\therefore t = \frac{25}{2}\cdot\log\left(\frac{P}{P_0}\right)$$

When $P = 2P_0$

$$t = \frac{25}{2}\cdot\log\left(\frac{2P_0}{P_0}\right) = \frac{25}{2}\log 2 = 8.64$$

Question264

$y = c^2 + \frac{c}{x}$ is the solution of the differential equation MHT CET 2020 (14 Oct Shift 1)

Options:

A. $x^4\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) - y = 0$

B. $x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) - y = 0$

C. $x^4\left(\frac{dy}{dx}\right)^2 - x\left(\frac{dy}{dx}\right) + y = 0$

D. $x^4\left(\frac{dy}{dx}\right)^2 + x\left(\frac{dy}{dx}\right) + y = 0$

Answer: B

Solution:

Given $y = c^2 + \frac{c}{x}$ Differentiating w.r.t. x

$$\frac{dy}{dx} = 0 - \frac{c}{x^2} = -\frac{c}{x^2} \Rightarrow c = (-x^2)\left(\frac{dy}{dx}\right)$$

Substituting value of c in given equation, We get

$$y = \left[(-x^2)\left(\frac{dy}{dx}\right)\right]^2 + \frac{(-x^2)\left(\frac{dy}{dx}\right)}{x}$$

$$= x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx}$$

$$\therefore x^4\left(\frac{dy}{dx}\right)^2 - x\frac{dy}{dx} - y = 0$$

Question265

Bacteria increases at the rate proportional to the number of bacteria present. If theoriginal number N doubles in 4 hours, then the number of bacteria will be $4\ \mathrm{N}$ in MHT CET 2020 (14 Oct Shift 1)

Options:

A. 2 hours

B. 4 hours

C. 6 hours

D. 8 hours

Answer: D

Solution:

Let N be the number of bacteria present at time t_0. Let $\mathrm{N_0}$ be the initial number of bacteria. Here
$\frac{\mathrm{dN}}{\mathrm{dt}} \alpha \mathrm{N} \Rightarrow \frac{\mathrm{dN}}{\mathrm{dt}} = \mathrm{KN} \Rightarrow \frac{\mathrm{dN}}{\mathrm{N}} = \mathrm{Kdt}$

$\therefore \int \frac{\mathrm{dN}}{\mathrm{N}} = \mathrm{K} \int \mathrm{dt} \Rightarrow \log \mathrm{N} = \mathrm{Kt} + \mathrm{C}$

When $\mathrm{t} = 0,\ \mathrm{N} = \mathrm{N_0}$

$$\therefore \log N_0 = C \Rightarrow \log\left(\frac{N}{N_0}\right) = Kt$$

When $t = 4, N = 2N_0 \Rightarrow 4K = \log 2$

$$\mathrm{K} = \frac{1}{4} \log 2$$
$$\therefore \log\left(\frac{\mathrm{N}}{\mathrm{N_0}}\right) = \frac{t}{4} \log 2$$

When $\mathrm{N} = 4\ \mathrm{N_0}$, we get

$$\log 4 = \frac{t}{4} \log 2 \Rightarrow 2(\log 2) = \frac{t}{4}(\log 2) \Rightarrow t = 8 \text{ hours}$$

This problem can also be saved as follows :

Number of bacteria doubles in 4 hrs.

$\therefore$ If initial number of bacteria are N, then

After 4 hours number of bacteria become $2\ \mathrm{N}$.

After 8 hours, number of bacteria becomes $4\ \mathrm{N}$

Question266

The general solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is MHT CET 2020 (14 Oct Shift 1)

Options:

A. $y^2 + 2\sin^{-1} x = c$

B. $x + \sin^{-1} y = c$

C. $y + \sin^{-1} x = c$

D. $x^2 + 2\sin^2 y = c$

Answer: C

Solution:

Given differential equation:

$$\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$$

Step 1: Separate variables

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

Step 2: Integrate both sides

$$\int dy = -\int \frac{1}{\sqrt{1-x^2}}\,dx$$

We know that:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x$$

So,

$$y = -\sin^{-1}x + c$$

Step 3: Simplify the expression

$$y + \sin^{-1}x = c$$

🧠 **Shortcut Tip:**

Whenever you see

$$\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$$

→ directly integrate to get

$$y + \sin^{-1}x = c$$

✅ **Final Answer:**

$$\boxed{y + \sin^{-1}x = c}$$

Question267

If the radius of a circle increases at the rate of 7 cm/sec, then the rate of increase ofits area after 10 minutes is MHT CET 2020 (14 Oct Shift 1)

Options:

A. $1,84,800\ \mathrm{cm}^2/\mathrm{sec}$

B. $1,64,800\ \mathrm{cm}^2/\mathrm{sec}$

C. $1,88,400 \mathrm{~cm}^2/\mathrm{sec}$

D. $1,68,400 \mathrm{~cm}^2/\mathrm{sec}$

Answer: A

Solution:

$$\begin{aligned} \frac{d\varepsilon}{dt} &= 7 \mathrm{~cm}/\mathrm{sec.} \\ A &= \pi\varepsilon^2 \\ \frac{dA}{dt} &= 2\pi\varepsilon\frac{d\varepsilon}{dt} - (1) \\ \text{Radius after 10 minutes };\varepsilon &= 70 \mathrm{~cm} \times 60 \mathrm{~cm} \\ \frac{dA}{dt} &= 2 \times \frac{22}{7} \times 70 \times 7 \times 60[\text{Put in }(1)\ldots \\ &= 1,84,800 \mathrm{~cm}^2/\mathrm{sec.} \end{aligned}$$

Question268

If the population grows at the rate of 5% per year, then the time taken for thepopulation to become double is (Given $\log 2 = 0.6912$) MHT CET 2020 (13 Oct Shift 2)

Options:

A. 13.624 years

B. $13 \cdot 8240$ years

C. 13.725 years

D. 13.8275 years

Answer: B

Solution:

Let P be the population at time t and P_0 be the initial population.

$$\text{Given } \frac{dP}{dt}=\frac{5P}{100} \Rightarrow \int \frac{dP}{P}=\int \frac{1}{20} dt$$

$$\log P=\frac{1}{20} t+c \ldots(1)$$

We have $t=0, P=P_0$

$\therefore \log P_0=c$

$\therefore \log P \quad=\frac{t}{20}+\log P_0$

$\therefore \log \left(\frac{P}{P_0}\right)=\frac{t}{20}$ ...(2)

When $P=2 P_0$, we write

$\log \left(\frac{2 P_0}{P_0}\right)=\frac{t}{20} \Rightarrow \log 2=\frac{t}{20}\ t=20(0.6912)=13.824$ years

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Question269

The particular solution of the differential equation $x d y+2 y d x=0$, when $x=2$and $y=1$ is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $x y^2=4$

B. $x^2 y=4$

C. $x^2 y=-4$

D. $x y^2=-4$

Answer: B

Solution:

Given D.E. is $x d y+2 y d x=0$

$\therefore x d y=-2 y d x \Rightarrow \int \frac{d y}{y}=\int-\frac{2 d x}{x}$

$\log y=-2 \log x+\log c \Rightarrow \log y+2 \log x=\log c$

$\therefore \quad \log y+\log x^2=\log c \Rightarrow x^2 y=c$

when $x=2, y=1, c=4$

$\therefore$ Particular solution is $x^2 y=4$

Question270

A metal has half life period of 10 days, A sample originally has a mass of $1000\mathrm{mg}$,then the mass remaining after 50 days is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $\frac{225}{8} mg$

B. $\frac{125}{8}\mathrm{mg}$

C. $\frac{125}{4}\mathrm{mg}$

D. $\frac{225}{4} mg$

Answer: C

Solution:

A mass has half life period of 10 days. It means every ten days, mass remaining is half of the mass before 10 days.

Initial mass $= 1000\mathrm{mg}$.

$\therefore$ Mass after, 10 days $= \frac{1}{2} \times 1000 = 500\mathrm{mg}$

Mass after 20 days $= \frac{1}{2} \times 500 = 250\mathrm{mg}$

Mass after 30 days $= \frac{1}{2} \times 250 = 125\mathrm{mg}$

Mass after 40 days $= \frac{1}{2} \times 125 = \frac{125}{2}\mathrm{mg}$

Mass after 50 days $= \frac{1}{2} \times \frac{125}{2} = \frac{125}{4}\mathrm{mg}$

Question271

The solution of $rdx + \left(x - r^2\right) dr = 0$ is MHT CET 2020 (13 Oct Shift 2)

Options:

A. $r^2 x = \frac{r^3}{3} + c$

B. $rx = \frac{r^2}{2} + c$

C. $x = \frac{r^3}{3} + c$

D. $rx = \frac{r^3}{3} + c$

Answer: D

Solution:

Given $\mathrm{rdx}+\left(\mathrm{x}-\mathrm{r}^{2}\right) \mathrm{dr}=0 \Rightarrow \mathrm{rdx}=-\left(\mathrm{x}-\mathrm{r}^{2}\right) \mathrm{dr}$

$\therefore \mathrm{r} \frac{\mathrm{dx}}{\mathrm{dr}}=\mathrm{r}^{2}-\mathrm{x} \Rightarrow \mathrm{r} \frac{\mathrm{dx}}{\mathrm{dr}}+\mathrm{x}=\mathrm{r}^{2}$

$\therefore \frac{\mathrm{dx}}{\mathrm{dr}}+\frac{\mathrm{x}}{\mathrm{r}}=\mathrm{r}$

$\text{I.F. }=e^{\int \frac{1}{r} dr}=e^{\log r}=r$

Solution is $\mathrm{x} \cdot \mathrm{r}=\int \mathrm{r} \cdot \mathrm{rdr}+\mathrm{c}$

$\mathrm{xr}=\frac{\mathrm{r}^{3}}{3}+\mathrm{c}$

Question272

$\tan^{-1} x+\tan^{-1} y=c$ is the general solution of the differential equation MHT CET 2020 (13 Oct Shift 2)

Options:

A. $\frac{dy}{dx}=-\left(\frac{1+y^{2}}{1+x^{2}}\right)$

B. $\frac{dy}{dx}=\left(\frac{1+y^{2}}{1+x^{2}}\right)$

C. $\frac{dy}{dx}=-\left(\frac{1+x^{2}}{1+y^{2}}\right)$

D. $\frac{dy}{dx}=\left(\frac{1+x^{2}}{1+y^{2}}\right)$

Answer: A

Solution:

Given $\tan^{-1} x+\tan^{-1} y=c$

$\therefore \frac{1}{1+x^{2}}+\frac{1}{1+y^{2}} \frac{dy}{dx}=0$

$\frac{1}{1+y^{2}} \frac{dy}{dx}=-\frac{1}{1+x^{2}} \quad \Rightarrow \frac{dy}{dx}=-\left(\frac{1+y^{2}}{1+x^{2}}\right)$

Question273

In a certain culture of bacteria, the rate of increase is proportional to the numberpresent. It is found that the number doubles in 4 hours. Then the number of timesthe bacteria are increased in 12 hours is MHT CET 2020 (13 Oct Shift 1)

Options:

A. 6

B. 8

C. 12

D. 4

Answer: B

Solution:

Let x be the number of bacteria in certain culture at time t. Then rate of increase is $\frac{dx}{dt}$ which is proportional to x.

$\therefore \frac{dx}{dt} \propto x \Rightarrow \frac{dx}{dt} = Kx \Rightarrow \frac{dx}{x} = Kdt \Rightarrow \int \frac{dx}{x} = K \int dt$
$\therefore \log x = Kt + c \ldots (1)$
Initially when $t = 0$, let $x = x_0$
$\therefore \log x = 0 + c \Rightarrow c = \log x_0$
$\therefore$ from (1) $\log x = kt + \log x_0$
$\therefore \log\left(\frac{x}{x_0}\right) = Kt \ldots (2)$

Given number doubles in 4 hrs i.e. when $t = 4, x = 2x_0 \therefore \log\left(\frac{2x_0}{x_0}\right) = 4K \Rightarrow K = \frac{1}{4}\log 2$

$\therefore$ from (2) $\log\left(\frac{x}{x_0}\right) = \frac{t}{4}\log 2$

When $t = 12$

$\log\left(\frac{x}{x_0}\right) = \frac{12}{4}\log 2 = 3\log 2 = \log 2^3 = \log 8$

$\therefore \frac{x}{x_0} = 8 \Rightarrow x = 8x_0$

This problem can be alternatively solved as follows :

Let the intial number of bacteria be x.

$\therefore$ Number of bacteria after 4hrs $= 2x$

Number of bacteria after 8hrs $= 2(2x) = 4x$

Number of bacteria after 12hrs $= 2(4x) = 8x$

Question274

The integrating factor of the differential equation $\frac{dy}{dx} + \frac{1}{x}y = x^3 - 3$ is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $-y$

B. y

C. x

D. $-x$

Answer: C

Solution:

$\frac{dy}{dx}+\frac{1}{x}y=x^3-3$ is linear differentiatial equation

$\therefore$ I.F. $=e^{\int\frac{1}{x}dx}=e^{\log x}=x$

Question275

The general solution of the differential equation $\sec^2 x$ tany $dx+\sec^2 y\tan x dy=0$is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\tan x\tan y=c$

B. $\sec x\tan y=c$

C. $\sec x\sec y=c$

D. $\tan x\sec y=c$

Answer: A

Solution:

$$\sec^2 x\tan y dx+\sec^2 y\tan x dy=0$$
$$\therefore \sec^2 x\tan y dx=-\sec^2 y\tan x dy$$
$$\frac{\sec^2 x}{\tan x}dx=-\frac{\sec^2 y}{\tan y}dy\Rightarrow\int\frac{\sec^2 x}{\tan x}dx=-\int\frac{\sec^2 y}{\tan y}dy$$
$$\log(\tan x)=-\log(\tan y)+\log c$$
$$\therefore \log(\tan x\tan y)=\log c$$
$$\tan x\tan y=c$$

Question276

Water at 100°C cools in 15 minutes to 75°C in a room temperature of 25°C. Then thetemperature of water after 30 minutes is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\left(\frac{400}{9}\right)^\circ$C

B. $\left(\frac{526}{9}\right)^\circ$C

C. $\left(\frac{335}{9}\right)^{\circ}$C

D. $\left(\frac{175}{3}\right)^{\circ}$C

Answer: D

Solution:

Let $\theta^{\circ}C$ be the temperature of water at time t min. Room temperature is given $25^{\circ}C$. then by Newton's law of cooling $\frac{-d\theta}{dt} \propto (\theta - 25)$

$\therefore \frac{d\theta}{dt} = -K(\theta - 25)$ where $K > 0$

$\therefore \frac{d\theta}{\theta - 25} = -Kdt \Rightarrow \int \frac{d\theta}{\theta - 25} = -K \int dt$

$\log(\theta - 25) = -Kt + c \ldots (1)$

When $t = 0, \theta = 100^{\circ}$

$\therefore \log 75 = c$

$\therefore$ from (1) $\log(\theta - 25) = -Kt + \log 75$

$\therefore \log\left(\frac{\theta - 25}{75}\right) = -Kt \ldots (2)$

When $t = 15, \theta = 75^{\circ}$

$$\log\left(\frac{50}{75}\right) = -15\,K \Rightarrow K = \frac{-1}{15}\log\left(\frac{2}{3}\right)$$
$$\therefore \log\left(\frac{\theta - 25}{75}\right) = \frac{t}{15}\log\left(\frac{2}{3}\right)\ldots[from 2]$$
$$\text{When } t = 30^{\circ} \log\left(\frac{\theta - 25}{75}\right) = \frac{30}{15}\log\left(\frac{2}{3}\right)$$
$$\log\left(\frac{\theta - 25}{75}\right) = 2\log\left(\frac{2}{3}\right) = \log\left(\frac{2}{3}\right)^2 = \log\left(\frac{4}{9}\right)$$
$$\therefore \frac{\theta - 25}{75} = \frac{4}{9} \Rightarrow \theta = \frac{525}{9} = \left(\frac{175}{3}\right)^{\circ}C$$

Question277

The equation of the curve which passes through point $(1, 0)$ and has tangent withslope $1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$ is MHT CET 2020 (13 Oct Shift 1)

Options:

A. $\tan^{-1}\left(\frac{x}{y}\right) = \log|x|$

B. $\tan^{-1}\left(\frac{x}{y}\right) = \log|y|$

C. $\tan^{-1}\left(\frac{y}{x}\right)=\log|y|$

D. $\tan^{-1}\left(\frac{y}{x}\right)=\log|x|$

Answer: D

Solution:

We have siope of tangent $=\frac{dy}{dx}=1+\frac{y}{x}+\left(\frac{y}{x}\right)^2$

$\therefore \frac{dy}{dx}=1+\frac{y}{x}+\frac{y^2}{x^2} \Rightarrow \frac{dy}{dx}=\frac{x^2+xy+y^2}{x^2}$ ...(1)

Put $y=ux \Rightarrow \frac{dy}{dx}=u+x\frac{du}{dx}$

$\therefore \quad u+x\frac{du}{dx}=\frac{x^2+ux^2+u^2x^2}{x^2}$

$\therefore \quad u+x\frac{du}{dx}=1+u+u^2 \Rightarrow x\frac{du}{dx}=1+u^2 \Rightarrow \int\frac{du}{1+u^2}=\int\frac{dx}{x}$

$\therefore \tan^{-1}u=\log|x|+c \Rightarrow \tan^{-1}\left(\frac{y}{x}\right)=\log|x|+c$

At $(1,0)$, we write $\tan^{-1}(0)=\log|1|+c \Rightarrow c=0$

$\therefore \tan^{-1}\left(\frac{y}{x}\right)=\log|x|$

Question278

The rate of growth of bacteria is proportional to the bacteria present. If it is foundthat the number doubles in 3 hours, then the number of times the bacteria areincreased in 6 hours is MHT CET 2020 (12 Oct Shift 2)

Options:

A. 6 times the original

B. 4 times the original

C. 8 times the original

D. 5 times the original

Answer: B

Solution:

Let b be the number of bacteria.

We have $\frac{\mathrm{db}}{\mathrm{dt}} \propto \mathrm{b} \Rightarrow \int \frac{\mathrm{db}}{\mathrm{b}} = \int \mathrm{Kdt}$

$\therefore \log \mathrm{b} = \mathrm{Kt} + \mathrm{c}$ ...(1)

Let b_0 be the initial number of bacteria. At $t = 0, b = b_0$

$\log \mathrm{b}_0 = \mathrm{K}(0) + \mathrm{c} \Rightarrow \mathrm{c} = \log \mathrm{b}_0$

$\therefore \log\left(\frac{b}{b_0}\right) = \mathrm{Kt}$ ...(2)

When $t = 3, b = 2b_0$

$\therefore \log\left(\frac{2b_0}{b_0}\right) = 3K \Rightarrow K = \frac{1}{3}(\log 2)$

Thus $\log b = \frac{1}{3}(\log 2)t + \log b_0$

When $\mathrm{t} = 6$

$\log\left(\frac{b}{b_0}\right) = 2\log 2 = \log 4 \Rightarrow \frac{b}{b_0} = 4 \Rightarrow b = 4b_0$

This problem can also be solved as follows :

The number of bacteria doubles in 3 hours.

Let initial number of bacteria $= \mathrm{N}$.

$\therefore$ After 3 hours, number of bacteria $= 2\ \mathrm{N}$.

After 6 hours, number of bacteria $= 4\ \mathrm{N}$.

Question279

The differential equation whose solution is $y = e^{ax}$ is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $y\frac{dy}{dx} = x\log y$

B. $\frac{dy}{dx} = x\log x$

C. $\frac{dy}{dx} = y\log x$

D. $x\frac{dy}{dx} = y\log y$

Answer: D

Solution:

Given $y = e^{ax}$

Taking log on both sides, we get

$\therefore \frac{\log y}{y} = \log e^{ax} \Rightarrow \log y = \text{ax} \log e \Rightarrow \log y = ax$ ...(1)

$\therefore \frac{1}{y} \frac{dy}{dx} = a$

Substituting value of ' a ' in equation (1), we get

$\log y = \left[\left(\frac{1}{y}\right) \frac{dy}{dx}\right] x \Rightarrow x \frac{dy}{dx} = y \log y$

Question280

A body cools according to Newton's law from $100^{\circ}\mathrm{C}$ to $60^{\circ}\mathrm{C}$ in 20 minutes. Thetemperature of the surrounding being $20^{\circ}\mathrm{C}$ then the temperature of the body afterone hour is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $15^{\circ}\mathrm{C}$

B. $30^{\circ}\mathrm{C}$

C. $40^{\circ}\mathrm{C}$

D. $20^{\circ}\mathrm{C}$

Answer: B

Solution:

Let $\theta^{\prime} C$ be the temperature of the body at time t. The temperature of surrounding is $20^{\circ} C$.

According to Newton's law of cooling

$\frac{d\theta}{dt} \propto (\theta - 20) \Rightarrow \frac{d\theta}{dt} = -K(\theta - 20)$, where $K > 0$

$\therefore \int \frac{d\theta}{\theta - 20} = \int -K dt \Rightarrow \log|\theta - 20| = -Kt + c$

We have $\theta = 100$ and $t = 0$

$\therefore \log|100 - 20| = 0 + c \Rightarrow c = \log 80$

$\therefore \log|\theta - 20| = -\mathrm{Kt} + \log 80$

$\therefore \log\left|\frac{\theta - 20}{80}\right| = -\mathrm{Kt}$

When $t = 20, \theta = 60$

$\therefore K = \frac{-1}{20} \log\left(\frac{1}{2}\right)$

Thus $\log\left(\frac{\theta - 20}{80}\right) = \frac{t}{20} \log\left(\frac{1}{2}\right)$

When $t = 60$

$\log\left(\frac{\theta - 20}{80}\right) = 3 \log\left(\frac{1}{2}\right) = \log\left(\frac{1}{8}\right) \Rightarrow \frac{\theta - 20}{80} = \frac{1}{8} \Rightarrow \theta = 30^{\circ}$

Question281

The differential equation obtained by eliminating the arbitrary constants from theequation $y^2 = (2x + c)^5$ is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\left(\frac{dy}{dx}\right)^4 - 625y^4 = 0$

B. $\left(\frac{dy}{dx}\right)^5 - 3125y^3 = 0$

C. $\left(\frac{dy}{dx}\right)^3 - 125y^3 = 0$

D. $xy\frac{dy}{dx} = 5$

Answer: B

Solution:

Given equation:

$$y^2 = (2x + c)^5$$

Step 1: Differentiate both sides w.r.t. x

$$2y\frac{dy}{dx} = 5(2x + c)^4 \times 2$$

$$\Rightarrow y\frac{dy}{dx} = 5(2x + c)^4$$

Step 2: From the original equation,

$$(2x + c)^5 = y^2$$

$$\Rightarrow (2x + c) = y^{\frac{2}{5}}$$

Step 3: Substitute this value into the differentiated equation

$$y\frac{dy}{dx} = 5\left(y^{\frac{2}{5}}\right)^4$$

$$\Rightarrow y\frac{dy}{dx} = 5y^{\frac{8}{5}}$$

$$\Rightarrow \frac{dy}{dx} = 5y^{\frac{8}{5}-1} = 5y^{\frac{3}{5}}$$

Step 4: Raise both sides to power 5

$$\left(\frac{dy}{dx}\right)^5 = 5^5 y^3$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^5 - 3125y^3 = 0$$

✅ **Final Answer:**

$$\boxed{\left(\frac{dy}{dx}\right)^5 - 3125y^3 = 0}$$

Question282

The differential equation of the family of lines having x - intercept ' a ' and y - intercept ' b ' is MHT CET 2020 (12 Oct Shift 2)

Options:

A. $\frac{d^2y}{dx^2} = -1$

B. $\frac{d^2y}{dx^2} = 10$

C. $\frac{d^2y}{dx^2} = 1$

D. $\frac{d^2y}{dx^2} = 0$

Answer: D

Solution:

Equation of line having $x-$ intercept a and y intercept b is $\frac{x}{a} + \frac{y}{b} = 1$ i.e. $bx + ay = ab$ Differentiating w.r.t. x, we get

$\mathrm{b} + \mathrm{a}\frac{\mathrm{dy}}{\mathrm{dx}} = 0 \Rightarrow \mathrm{a}\frac{\mathrm{dy}}{\mathrm{dx}} = -\mathrm{b} \Rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{-\mathrm{b}}{\mathrm{a}}$

Again differentiating w.r.t. $\mathbf{x}$, we get

$\frac{d^2y}{dx^2} = 0$

Question283

Degree of the differential equation $e^{\frac{dy}{dx}} + \left(\frac{dy}{dx}\right)^3 = x$ is MHT CET 2020 (12 Oct Shift 1)

Options:

A. 2

B. 1

C. not defined

D. 3

Answer: C

Solution:

The degree of differential equation is the power of the highest ordered derivative when te: derivatives are made free from fractional or negative indices if any.

So here degree is not defined.

Question284

The solution of differential equation $x^2\frac{dy}{dx} = y^2 + xy$ is MHT CET 2020 (12 Oct Shift 1)

Options:

A. $\frac{x}{y} + \log|x| = c$

B. $\frac{y}{x} + \log|x| = c$

C. $\frac{x}{y} - \log|x| = c$

D. $\frac{y}{x} - \log|x| = c$

Answer: A

Solution:

We have $x^2 \frac{dy}{dx} = y^2 + xy$

$\therefore \frac{dy}{dx} = \frac{y^2+xy}{x^2} \ldots (1)$

$\therefore$ Put $y = ux$

$\therefore \frac{dy}{dx} = u + x\frac{du}{dx}$

$\therefore$ Equation (1) becomes

$u + x\frac{du}{dx} = \frac{u^2x^2 + x(ux)}{x^2} \Rightarrow u + x\frac{du}{dx} = u^2 + u$

$\therefore x\frac{du}{dx} = u^2$

$\therefore \int \frac{du}{u^2} = \int \frac{dx}{x}$

$\therefore \frac{-1}{u} = \log|x| + c_1 \Rightarrow \quad \frac{-x}{y} = \log|x| + c_1$

$\therefore \frac{x}{y} + \log|x| = c$

Question285

The particular solution of the differential equation $\sin^2 y \frac{dx}{dy} + x = \cot y$ **when** $x = 0$ **and** $y = \frac{3\pi}{4}$ **is MHT CET 2020 (12 Oct Shift 1)**

Options:

A. $x = 1 + \cot y$

B. $xy = \cot(x + y)$

C. $xy = \cot(x - y)$

D. $y = 1 + \cot x$

Answer: A

Solution:

We have,

$\sin^2 y \frac{dx}{dy} + x = \cot y$

$\therefore \frac{dx}{dy} + (\operatorname{cosec}^2 y) x = \cot y \cdot \operatorname{cosec}^2 y$

$\therefore$ I.F. $= e^{\int \operatorname{cosec}^2 y dy} = e^{-\cot y}$

$\therefore x e^{-\cot y} = \int e^{-\cot y} \cot y \cos ec^2 y dy$

In RHS, put $-\cot y = t \Rightarrow \operatorname{cosec}^2 y dy = dt$

$\therefore \quad x e^{-\cot y} = \int e^t(-t) dt = -\int t e^t dt$

$= -\left[t e^t - \int e^t dt\right] = -t e^t + e^t = e^t(1-t)$

$x e^{-\cot y} = e^{-\cot y}(1+\cot y) + C$...(2)

At $x = 0, y = \frac{3\pi}{4}$, we get $0 = e(1-1) + C$

From equation (2), required solution is $x e^{-\cot y} = e^{-\cot y}(1+\cot y)$

$x \quad = 1 + \cot y \, x = 1 + \cot y$

Question286

The rate of growth of bacteria is proportional to number present. If initially therewere 1000 bacteria and the number doubles in 1 hour then the number of bacteriaafter $2\frac{1}{2}$ hours are (Given $\sqrt{2} = 1.414$) MHT CET 2020 (12 Oct Shift 1)

Options:

A. 4646 approximately

B. 5056 approximately

C. 5656 approximately

D. $400\sqrt{2}$ approximately

Answer: C

Solution:

The rate of growth is proportional to the number present

$\therefore \frac{dN}{dt} \propto N \Rightarrow \frac{dN}{dt} = kN \Rightarrow \frac{dN}{N} = kdt$

$\therefore$ On integrating we get

$\int \frac{dN}{N} = k \int dt$

$\therefore \log N = kt + C$

When $t = 0,\ N = 1000 \Rightarrow C = \log 1000$

$\therefore \log N = kt + \log 1000$

$\therefore \quad \log\left(\frac{N}{1000}\right) = kt$

$N = 1000e^{kt}$ ...(1)

When $t = 1,\ N = 2000$

$\therefore e^k = 2 \quad \Rightarrow N = 1000 \times 2^1 \quad \ldots$[from (1)]

When $t = 2\frac{1}{2}$, we get

$N = 1000 \times 2^{\frac{5}{2}} = 1000 \times 4\sqrt{2}$

$= 1000 \times 4 \times 1.414 = 5656$

Question287

The order of the differential equation of all circles whose radius is 4, is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. 1

B. 2

C. 3

D. 4

Answer: B

Solution:

Let general equation of circle is
$x^2 + y^2 + 2gx + 2fy + c = o$
Its radius, $\sqrt{g^2 + f^2 - c} = 4$

Then, $c=g^2+f^2-16$
Hence, we have 2 necessary arbitrary constants g and f. Then, order of differential equation is 2.

Question288

The general solution of $x\frac{dy}{dx}=y-x\tan\left(\frac{y}{x}\right)$ is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $x^2\sin\left(\frac{x}{y}\right)=c$

B. $x\sin\left(\frac{x}{y}\right)=c$

C. $x\sin\left(\frac{y}{x}\right)=c$

D. $x^2\sin\left(\frac{y}{x}\right)=c$

Answer: C

Solution:

Given equation is
$\frac{dy}{dx}=\frac{y}{x}-\tan\frac{y}{x}$
is Homogeneous equation, then
Put $y=vx$ and $\frac{dy}{dx}=v+x\frac{dv}{dx}$
Then,
$v+x\frac{dv}{dx}=v-\tan v$
$\Rightarrow\frac{dv}{\tan v}=-\frac{dx}{x}$ (integrate both sides)
$\ln\sin v=-\ln x+\ln c$
$\ln\left(x\cdot\sin\frac{y}{x}\right)=\ln c\Rightarrow x\sin\frac{y}{x}=c$

Question289

The general solution of the differential equation of all circles having centre at $A(-1,2)$ is ________ MHT CET 2019 (02 May Shift 1)

Options:

A. $x^2+y^2+x-2y+c=0$

B. $x^2+y^2-2x+4y+c=0$

C. $x^2+y^2-x+2y+c=0$

D. $x^2+y^2+2x-4y+c=0$

Answer: D

Solution:

Equation of circle is $x^2+y^2+2x-4y+c=0$
Whose centre is $(-1,2)$

Question290

The particular solution of the differential equation log $\left(\frac{dy}{dx}\right)=x$, when x =0, y = 1 is ….. MHT CET 2019 (Shift 2)

Options:

A. $y=e^x+2$

B. $y=-e^x$

C. $y=-e^x+2$

D. $y=e^x$

Answer: D

Solution:

We have, differential equations,
$\log\left(\frac{dy}{dx}\right)=x \Rightarrow \frac{dy}{dx}=e^x$
$\Rightarrow dy=e^x dx$
Integrating on both sides, we get
$\int dy=\int e^x dx$
$\Rightarrow y=e^x+C \ldots$ (i)
On putting x = 0, y = 1 is Eq. (i), we get
$1=e^0+C \Rightarrow C=0$
Now, particular solution of the given differential is $y=e^x$

Question291

The solution of the differential equation $ydx-xdy=xydx$ is …. MHT CET 2019 (Shift 2)

Options:

A. $x^2 = e^x y^2$

B. $x = ye^x$

C. $xy = e^x$

D. $x^2 y^2 = \log x$

Answer: B

Solution:

We have differential equation $ydx - xdy = xydx$
$\Rightarrow \frac{ydx - xdy}{xy} = dx$
$\Rightarrow d\left(\log\frac{x}{y}\right) = dx$
On integrating both sides, we get
$\log\left(\frac{x}{y}\right) = x \Rightarrow \frac{x}{y} = e^x$
$\Rightarrow x = ye^x$

Question292

The solution of the differential equation $\frac{d\theta}{dt} = -k(\theta - \theta_0)$, where k is constant, is ...... MHT CET 2019 (Shift 2)

Options:

A. $\theta = \theta_0 + ae^{-kt}$

B. $\theta = \theta_0 + ae^{kt}$

C. $\theta = 2\theta_0 - ae^{kt}$

D. $\theta = 2\theta_0 - ae^{-kt}$

Answer: A

Solution:

We have differential equation
$\frac{d\theta}{dt} = -k(\theta - \theta_0)$, where k is constant
$\Rightarrow \frac{d\theta}{dt} + k\theta = k\theta_0$
Which is linear differential equation in form of
$\frac{dy}{dx} + Py = Q$
$\therefore IF =_e \int kdt = e^{kt}$

Therefore, required solution,
$(\theta)\left(e^{kt}\right)=\int\left(e^{kt}\times k\theta_0\right)dt$
$\Rightarrow \theta^{ekt}=e^{kt}\theta_0+a$
$\Rightarrow \theta=\theta_0+ae^{-kt}$

Question293

The order of the differential equation of all circles which lie in the first quadrant and touch both the axes is… MHT CET 2019 (Shift 1)

Options:

A. Two

B. Three

C. One

D. Four

Answer: C

Solution:

Equation of the circle touching both axes is $x^2+y^2-2ax-2ay+a^2=0$
Here, number of arbitrary constant is one which is equal to the order of its differential equation.

Question294

The solution of differential equation $\left(x^2+1\right)\frac{dy}{dx}+\left(y^2+1\right)=0$ is…. MHT CET 2019 (Shift 1)

Options:

A. $x+y=c$

B. $\left(x^2+1\right)\left(y^2+1\right)=c$

C. $x^2=y^2+c$

D. $\tan^{-1}x+\tan^{-1}y=c$

Answer: D

Solution:

We have, differential equation

$\left(x^2+1\right)\frac{dy}{dx}+\left(y^2+1\right)=0$

$\Rightarrow \frac{dx}{x^2+1}=-\frac{dy}{y^2+1}$

On the integrating both sides, we get

$\int \frac{dx}{x^2+1}=-\frac{dy}{y^2+1}$

$\Rightarrow \tan^{-1}x=-\tan^{-1}y+C$

$\Rightarrow \tan^{-1}x+\tan^{-1}y=C$

Question295

The order of the differential equation of all parabolas, whose latus rectum is $4a$ and axis parallel to the $x-axis$, is MHT CET 2018

Options:

A. One

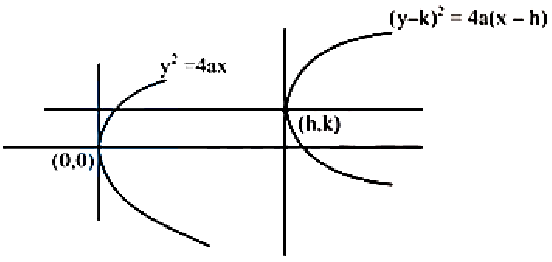
B. Four

C. Three

D. Two

Answer: D

Solution:



Since general equation of such parabola $(y-k)^2=4a(x-h)$ have two arbitrary constants h and k

$\therefore$ order $=2$

Question296

The general solution of differential equation $\frac{dy}{dx}=\cos(x+y)$ is MHT CET 2018

Options:

A. $\tan\left(\frac{x+y}{2}\right)=y+c$

B. $\tan\left(\frac{x+y}{2}\right)=x+c$

C. $\cot\left(\frac{x+y}{2}\right)=y+c$

D. $\cot\left(\frac{x+y}{2}\right)=x+c$

Answer: B

Solution:

$$\text{Let } x+y=v$$
$$1+\frac{dy}{dx}=\frac{dv}{dx}$$
$$\frac{dy}{dx}=\frac{dv}{dx}-1$$
$$\therefore \frac{dv}{dx}-1=\cos v$$
$$\Rightarrow \frac{dv}{dx}=1+\cos v$$
$$\Rightarrow \frac{dv}{dx}=2\cos^2\frac{v}{2}$$
$$\Rightarrow \int\frac{dv}{\cos^2\frac{v}{2}}=2\int dx$$
$$\Rightarrow \int\sec^2\left(\frac{v}{2}\right)\, dv=2x+c$$
$$\Rightarrow 2\tan\left(\frac{v}{2}\right)=2x+c$$
$$\Rightarrow 2\tan\left(\frac{x+y}{2}\right)=2x+c$$
$$\Rightarrow \tan\left(\frac{x+y}{2}\right)=x+\frac{c}{2}$$
$$\Rightarrow \tan\left(\frac{x+y}{2}\right)=x+c$$

Question297

If $y=\left(\tan^{-1}x\right)^2$ then $\left(x^2+1\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=$ MHT CET 2018

Options:

A. 4

B. 2

C. 1

D. 0

Answer: B

Solution:

$$y=\left(\tan^{-1}x\right)^2$$
$$\frac{dy}{dx}=\frac{2\tan^{-1}(x)}{(1+x^2)}$$
$$\left(1+x^2\right)\frac{dy}{dx}=2\tan^{-1}(x)$$
$$\left(1+x^2\right)\frac{d^2y}{dx^2}+\frac{dy}{dx}(2x)=\frac{2}{1+x^2}$$
$$\left(1+x^2\right)^2\frac{d^2y}{dx^2}+2x\left(x^2+1\right)\frac{dy}{dx}=2$$

Question298

The solution of the differential equation $\frac{dy}{dx}=\tan\left(\frac{y}{x}\right)+\frac{y}{x}$ is MHT CET 2017

Options:

A. $\cos\left(\frac{y}{x}\right)=cx$

B. $\sin\left(\frac{y}{x}\right)=cx$

C. $\cos\left(\frac{y}{x}\right)=cy$

D. $\sin\left(\frac{y}{x}\right)=cy$

Answer: B

Solution:

$$\frac{dy}{dx}=\tan\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)$$
$$\frac{y}{x}=v$$
$$y=vx$$
$$\frac{dy}{dx}=v+x\frac{dv}{dx}$$
$\therefore$ the given equation becomes
$$v+x\frac{dv}{dx}=\tan v+v$$
$$\frac{1}{\tan v}dv=\frac{1}{x}dx$$
$$\int\cot v\,dv=\int\frac{1}{x}dx$$
$$\log|\sin v|=\log x+\log c$$
$$=\log(xc)$$
$$\sin v=xc$$
$$\sin\left(\frac{y}{x}\right)=xc$$

Question299

The differential equation of all parabolas whose axis is $y-$ axis is MHT CET 2017

Options:

A. $x\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

B. $x\frac{d^2y}{dx^2}+\frac{dy}{dx}=0$

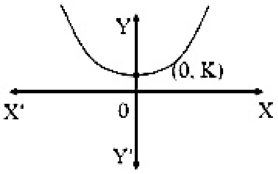
C. $\frac{d^2y}{dx^2}-y=0$

D. $\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

Answer: A

Solution:

Axis = y axis
Vertex is (0, k)
Equation of parabola is
$(x-0)^2=4a(y-k)$
$x^2=4ay-4ak$



Differentiate w.r.t x,
$2x=4a\frac{dy}{dx}$
$\Rightarrow x=2a\frac{dy}{dx}$
$\therefore \quad \frac{1}{2a}=\frac{1}{x}\frac{dy}{dx}$
Differentiate w.r.t x,
$\frac{d}{dx}\left(\frac{1}{x}\cdot\frac{dy}{dx}\right)=\frac{d}{dx}\left(\frac{1}{2a}\right)$
$\Rightarrow\frac{1}{x}\cdot\frac{d^2y}{dx^2}+\frac{dy}{dx}\left(-\frac{1}{x^2}\right)=0$
$\Rightarrow\therefore \quad x\frac{d^2y}{dx^2}-\frac{dy}{dx}=0$

Question300

The particular solution of the differential equation $xdy+2ydx=0$, when $x=2,\ y=1$ is MHT CET 2017

Options:

A. $xy=4$

B. $x^2y=4$

C. $xy^2 = 4$

D. $x^2y^2 = 4$

Answer: B

Solution:

$$xdy + 2ydx = 0$$
$$\Rightarrow \quad \frac{dy}{y} + \frac{2dx}{x} = 0$$
On integrating
$$\int \frac{dy}{y} + 2\int \frac{dx}{x} = C_1$$
$$\log y + 2\log x \ = \log C$$
$$\therefore x^2y = C$$
When $x = 2,\ y = 1,\ C = 4$

$\therefore$ Particular solution is $x^2y = 4$

Question301

The differential equation of the family of circles touching y-axis at the origin is MHT CET 2016

Options:

A. $\left(x^2 + y^2\right)\frac{dy}{dx} - 2xy = 0$

B. $x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

C. $\left(x^2 - y^2\right)\frac{dy}{dx} - 2xy = 0$

D. $\left(x^2 + y^2\right)\frac{dy}{dx} + 2xy = 0$

Answer: B

Solution:

As required circle touches y - axis at the origin.

$\therefore$ Let Center of the circle is $d(a, 0)$ and radius is a

$\therefore$ Equation of circle will be,
$$(x - a)^2 + (y - 0)^2 = a^2$$
$$\Rightarrow \quad x^2 - 2ax + a^2 + y^2 = a^2$$
$$\Rightarrow \quad x^2 + y^2 - 2ax = 0 \text{ .... (i)}$$
By differentiating above equation w.r.t x, we get
$$2x + 2y\frac{dy}{dx} - 2a = 0$$

$\therefore\ 2a = 2x + 2y\frac{dy}{dx}$ .... (ii)

From (i) and (ii),

$x^2 + y^2 - \left(2x + 2y\frac{dy}{dx}\right)x = 0$

$\Rightarrow\ x^2 + y^2 - 2x^2 - 2xy\frac{dy}{dx} = 0$

$\Rightarrow x^2 - y^2 + 2xy\frac{dy}{dx} = 0$

Question302

If $y = e^{m\sin^{-1}x}$ and $(1-x^2)\left(\frac{dy}{dx}\right)^2 = Ay^2$, then $A =$ ________ MHT CET 2016

Options:

A. m

B. $-m$

C. m^2

D. $-m^2$

Answer: C

Solution:

Given, $y = e^{m\sin^{-1}\ x}$

$\Rightarrow\ \frac{dy}{dx} = e^{m\sin^{-1}\ x} \cdot m \cdot \frac{1}{\sqrt{1-x^2}}$

$\Rightarrow\ \left(\frac{dy}{dx}\right)^2 = \frac{m^2y^2}{1-x^2}$

$\therefore\ \ (1-x^2)\left(\frac{dy}{dx}\right)^2 = m^2y^2$

$\therefore\ \ A = m^2$

Question303

The degree and order of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^3\right]^{\frac{7}{3}} = 7\left(\frac{d^2y}{dx^2}\right)$ respectively are MHT CET 2016

Options:

A. 3 and 7

B. 3 and 2

C. 7 and 3

D. 2 and 3

Answer: D

Solution:

The given differential equation can be written as

$$\left[1+\left(\frac{dy}{dx}\right)^3\right]^7=7^3\left(\frac{d^2y}{dx^2}\right)^3$$

$\Rightarrow$ order = 2, degree = 3

Question304

The particular solution of the differential equation $y(1+\log\ x)\frac{dx}{dy}-x\log\ x=0$ when y(e)=e2 is

MHT CET 2016

Options:

A. $y=ex\log x$

B. $ey=x\ \log x$

C. $xy=e\log x$

D. $y\log x=ex$

Answer: A

Solution:

$$y(1+\log x)\frac{dx}{dy}-x\log\ x=0$$

$$\Rightarrow\ \left(\frac{1+\log\ x}{x\log\ x}\right)dx=\frac{dy}{y}$$

Integrating on both side

$$\int\left(\frac{1+\log\ x}{x\log\ x}\right)dx=\int\frac{dy}{y}$$

$$\log\ (x\log\ x)=\log\ y+\log\ C$$

$$\Rightarrow\ \log\ (x\log\ x)=\log\ (y.\ c)$$

$$\therefore\ x\ \log\ x=y.c\ ......\ \text{(i)}$$

As $x=e$, $y=e^2$

$$\therefore\ e=e^2\ .c$$

$$\therefore\ c=\frac{1}{e}$$

Putting $c=\frac{1}{e}$ in equation (i) we get

$x \log x = \frac{y}{e}$
$\Rightarrow \quad y = ex \log x$

Question305

If $\sin x$ is the integration factor of the linear differential equation $\frac{dy}{dx} + Py = Q$, then P is MHT CET 2016

Options:

A. $\log \sin x$

B. $\cos x$

C. $\tan x$

D. $\cot x$

Answer: D

Solution:

Given.
I.F of $\frac{dy}{dx} + py = Q$ is $\sin x$
$\therefore \quad e^{\int p\, dx} = \sin x$
$\Rightarrow \quad \int P\, dx = \ln(\sin x)$
By anti - differentiation method, we will get
$P = \frac{d}{dx}[\ln(\sin x)]$
$= \frac{1}{\sin x} . \cos x$
$P = \cot x$

Question306

The differential equation of all circles which pass through the origin and whose centres lie on y -axis is MHT CET 2011

Options:

A. $\left(x^2 - y^2\right) \frac{dy}{dx} - 2xy = 0$

B. $\left(x^2 - y^2\right) \frac{dy}{dx} + 2xy = 0$

C. $\left(x^2 - y^2\right) \frac{dy}{dx} - xy = 0$

D. $\left(x^2 - y^2\right) \frac{dy}{dx} + xy = 0$

Answer: A

Solution:

Equation of a circle is

$$x^2+(y-a)^2=a^2$$
$$2x+2y\frac{dy}{dx}-2a\frac{dy}{dx}=0$$

Form Eqs. (i) and (ii)

$$\frac{dy}{dx}=\frac{2xy}{x^2-y^2}$$
$$\Rightarrow \quad \left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$$

Question307

If m and n are order and degree of the equation $\left(\frac{d^2y}{dx^2}\right)^5+4\cdot\frac{\left(\frac{d^2y}{dx^2}\right)^3}{\left(\frac{d^3y}{dx^3}\right)}+\left(\frac{d^3y}{dx^3}\right)=x^2-1$, then MHT CET 2011

Options:

A. $m=3, n=3$

B. $m=3, n=2$

C. $m=3, n=5$

D. $m=3, n=1$

Answer: B

Solution:

The given differential equation can be rewritten as

$$\left(\frac{d^2y}{dx^2}\right)^5\cdot\frac{d^3y}{dx^3}+4\left(\frac{d^2y}{dx^2}\right)^3+\left(\frac{d^3y}{dx^3}\right)^2$$
$$=\left(x^2-1\right)\frac{d^3y}{dx^3}\Rightarrow \text{ order }(m)=3$$

and $\quad$ degree $(n)=2$

Question308

The integrating factor of the differential equation $\frac{dy}{dx}(x \log x)+y=2 \log x$ is given by MHT CET 2011

Options:

A. e^x

B. $\log x$

C. $\log (\log x)$

D. x

Answer: B

Solution:

$$\frac{dy}{dx}(x \log x)+y=2 \log x$$
$$\Rightarrow \quad \frac{dy}{dx}+\frac{y}{x \log x}=\frac{2}{x}$$

Here, $P=\frac{1}{x \log x}, Q=\frac{2}{x}$

IF

$$=e^{\int P d x}=e^{\int \frac{d x}{x \log x}}$$

$=\log x$

Question309

The differential equation whose solution is $(x-h)^2+(y-k)^2=a^2$(a is a constant), is MHT CET 2011

Options:

A. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2 \frac{d^2 y}{dx^2}$

B. $\left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2 y}{dx^2}\right)^2$

C. $\left[1+\left(\frac{dy}{dx}\right)\right]^3=a^2\left(\frac{d^2 y}{dx^2}\right)^2$

D. None of the above

Answer: B

Solution:

Given, $(x-h)^2+(y-k)^2=a^2$ ...(i)

$$\Rightarrow \quad 2(x-h)+2(y-k)\frac{dy}{dx}=0$$
$$\Rightarrow \quad (x-h)+(y-k)\frac{dy}{dx}=0$$

Again differentiating

$$(y-k)=-\frac{1+\left(\frac{dy}{dx}\right)^2}{d^2y/dx^2}$$

Putting in Eq. (ii), we get

$$x-h=-(y-k)\frac{dy}{dx}$$
$$=\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{dy}{dx}}{\frac{d^2y}{dx^2}}$$

Putting in Eq. (i), we get

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2\left(\frac{dy}{dx}\right)^2}{\left(\frac{d^2y}{dx^2}\right)^2}$$
$$\Rightarrow \quad \left[1+\left(\frac{dy}{dx}\right)^2\right]^2\left[\left(\frac{dy}{dx}\right)^2+1\right]=a^2\left(\frac{d^2y}{dx^2}\right)^2$$
$$+\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^2}{\left(\frac{d^2y}{dx^2}\right)^2}=a^2$$
$$\Rightarrow \left[1+\left(\frac{dy}{dx}\right)^2\right]^3=a^2\left(\frac{d^2y}{dx^2}\right)^2$$

Question310

The differential equation of family of circles whose centre lies on x -axis, is MHT CET 2010

Options:

A. $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$

B. $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2-1=0$

C. $y\frac{d^2y}{dx^2}-\left(\frac{dy}{dx}\right)^2-1=0$

D. $y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$

Answer: D

Solution:

The equation of family of circle having centre at x -axis is $x^2+y^2-2ax=0$.

On differentiating, we get

$$2x+2y\frac{dy}{dx}-2a=0$$

Again, differentiating, we get

$$2+2\left[y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2\right]=0$$
$$\Rightarrow \quad y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$$

Question311

The solution of the differential equation $y(1+\log x)\frac{dx}{dy}-x\log x=0$ is MHT CET 2010

Options:

A. $x\log x=y+c$

B. $x\log x=yc$

C. $y(1+\log x)=c$

D. $\log x-y=c$

Answer: B

Solution:

Given differential equation is

$$y(1+\log x)\frac{dx}{dy}-x\log x=0$$

$$\begin{aligned}
&\Rightarrow \quad \int \frac{1+\log x}{x\log x}dx=\int \frac{dy}{y} \\
&\Rightarrow \quad \int \frac{1}{x\log x}dx+\int \frac{1}{x}dx=\int \frac{1}{y}dy \\
&\Rightarrow \quad \log(\log x)+\log x=\log y+\log c \\
&\Rightarrow \quad x\log x=yc
\end{aligned}$$

Question312

The order of the differential equation whose solution is $ae^x+be^{2x}+ce^{3x}+d=0$, is MHT CET 2010

Options:

A. 1

B. 2

C. 3

D. 4

Answer: D

Solution:

Since, this equation has 4 arbitrary constants a, b, c, d therefore, order of this differential equation is 4.

Question313

General solution of the differential equation $\frac{dy}{dx}=\frac{x+y+1}{x+y-1}$ is given by MHT CET 2009

Options:

A. $x+y=\log|x+y|+c$

B. $x-y=\log|x+y|+c$

C. $y=x+\log|x+y|+c$

D. $y=x\log|x+y|+c$

Answer: C

Solution:

$\frac{dy}{dx}=\frac{x+y+1}{x+y-1}$

Put $x+y=t$

$\Rightarrow\quad 1+\frac{dy}{dx}=\frac{dt}{dx}$

$$\begin{aligned}\Rightarrow\quad & \frac{dy}{dx}=\frac{dt}{dx}-1\\ \therefore\quad & \frac{dt}{dx}-1=\frac{t+1}{t-1}\\ \Rightarrow\quad & \frac{dt}{dx}=\frac{t+1+t-1}{t-1}\\ \Rightarrow\quad & \frac{dt}{dx}=\frac{2t}{t-1}\\ \Rightarrow\quad & \left(\frac{t-1}{2t}\right)dt=dx\\ \Rightarrow\quad & \left(\frac{1}{2}-\frac{1}{2t}\right)dt=dx\end{aligned}$$

On integrating, we get

$$\frac{1}{2}t-\frac{1}{2}\log t=x+c_1$$

$$\begin{aligned}\Rightarrow\quad t-\log t &= 2x+2c_1\\ \Rightarrow\quad x+y-\log(x+y) &= 2x+2c_1\\ \Rightarrow\quad y &= x+\log(x+y)+c\end{aligned}$$

Question314

The order and degree of the differential equation $\frac{d^2y}{dx^2}=\sqrt[3]{1-\left(\frac{dy}{dx}\right)^4}$ are respectively

MHT CET 2009

Options:

A. 2,3

B. 3,2

C. 2,4

D. 2,2

Answer: A

Solution:

Given differential equation is

$$\begin{aligned} \frac{d^2y}{dx^2} &= \sqrt[3]{1-\left(\frac{dy}{dx}\right)^4} \\ \Rightarrow \left(\frac{d^2y}{dx^2}\right)^3 &= 1-\left(\frac{dy}{dx}\right)^4 \\ \therefore \quad \text{Order} &= 2, \text{ degree } = 3 \end{aligned}$$

Question315

Form the differential equation of all family of lines $y = mx + \frac{4}{m}$ by eliminating the arbitrary constant ' m' ' is MHT CET 2009

Options:

A. $\frac{d^2y}{dx^2} = 0$

B. $x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} + 4 = 0$

C. $x\left(\frac{dy}{dx}\right)^2 + y\frac{dy}{dx} + 4 = 0$

D. $\frac{dy}{dx} = 0$

Answer: B

Solution:

$y = mx + \frac{4}{m}$

$\therefore \quad \frac{dy}{dx} = m$

From Eq. (i), we get

$$\begin{aligned} y &= x\left(\frac{dy}{dx}\right) + \frac{4}{(dy/dx)} \\ \Rightarrow y\left(\frac{dy}{dx}\right) &= x\left(\frac{dy}{dx}\right)^2 + 4 \\ \Rightarrow x\left(\frac{dy}{dx}\right)^2 &- y\frac{dy}{dx} + 4 = 0 \end{aligned}$$

Which is the required differential equation.

Question316

If $xy = \tan^{-1}(xy) + \cot^{-1}(xy)$, then $\frac{dy}{dx}$ is equalto MHT CET 2009

Options:

A. $\frac{y}{x}$

B. $-\frac{y}{x}$

C. $\frac{x}{y}$

D. $-\frac{x}{y}$

Answer: B

Solution:

$xy = \tan^{-1}(xy) + \cot^{-1}(xy) = \frac{\pi}{2}$

$\Rightarrow \quad xy = \frac{\pi}{2}$

$\therefore \quad x\frac{dy}{dx} + y = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-y}{x}$

Question317

The solution of the differential equation $\frac{dy}{dx} = \frac{x-y+3}{2(x-y)+5}$ is MHT CET 2008

Options:

A. $2(x-y) + \log(x-y) = x + c$

B. $2(x-y) - \log(x-y+2) = x + c$

C. $2(x-y) + \log(x-y+2) = x + c$

D. None of the above

Answer: C

Solution:

Given differential equation is

$$\frac{dy}{dx}=\frac{x-y+3}{2(x-y)+5}$$

Let $x-y=v \Rightarrow \frac{dy}{dx}=1-\frac{dv}{dx}$

$\therefore$

$$\frac{dv}{dx}=\frac{v+2}{2v+5}$$

$$\begin{aligned}&\Rightarrow \quad \int \tfrac{2v+5}{v+2}dv=\int dx\\&\Rightarrow \quad \int\left(2+\tfrac{1}{v+2}\right)dv=\int dx\end{aligned}$$

$\Rightarrow \quad 2v+\log(v+2)=x+c$

$\Rightarrow 2(x-y)+\log(x-y+2)=x+c$

Question318

The solution of the differential equation $\left(3xy+y^2\right)dx+\left(x^2+xy\right)dy=0$ is MHT CET 2008

Options:

A. $x^2\left(2xy+y^2\right)=c^2$

B. $x^2\left(2xy-y^2\right)=c^2$

C. $x^2\left(y^2-2xy\right)=c^2$

D. None of these

Answer: A

Solution:

Homogeneous equation can be written in the form ofPut

$$\frac{dy}{dx}=-\frac{3xy+y^2}{x^2+xy}$$

$$y=vx \text{ and } \frac{dy}{dx}=v+x\frac{dv}{dx}, \text{ we get}$$

$$v+x\frac{dv}{dx}=-\frac{3x^2v+x^2v^2}{x^2+x^2v}$$

$$\Rightarrow \quad x\frac{dv}{dx}=\frac{-2v(v+2)}{v+1}$$

$$\Rightarrow \quad \tfrac{1}{x}dx=-\tfrac{(v+1)}{2v(v+2)}dv$$

$$\Rightarrow \quad -\tfrac{2}{x}=-\left[\tfrac{1}{2(v+2)}+\tfrac{1}{2v}\right]dv$$

On integrating, we get

$$-2\log_e x=\frac{1}{2}\log(v+2)+\frac{1}{2}\log v-\log c$$

$$\Rightarrow v(v+2)x^4=c^2$$

$$\Rightarrow \frac{y}{x}\left(\frac{y}{x}+2\right)x^4=c^2 \qquad \left(\because v=\frac{y}{x}\right)$$

Hence, required solution is $\left(y^2+2xy\right)x^2=c^2$

Question319

The order and degree of the differential equation $\sqrt{\frac{dy}{dx}}-4\frac{dy}{dx}-7x=0$ are MHT CET 2008

Options:

A. 1 and 1/2

B. 2 and 1

C. 1 and 1

D. 1 and 2

Answer: D

Solution:

Given differential equation:

$$\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x = 0$$

Step 1: Identify the order

Here, the highest derivative is $\frac{dy}{dx}$.

👉 Therefore, **Order** = **1**

Step 2: Remove the radical (square root)

To find the **degree**, remove the square root by squaring both sides.

$$\left(\sqrt{\frac{dy}{dx}} - 4\frac{dy}{dx} - 7x\right)^2 = 0$$

Simplify:

$$\sqrt{\frac{dy}{dx}} = 4\frac{dy}{dx} + 7x$$

Square both sides:

$$\frac{dy}{dx} = (4\frac{dy}{dx} + 7x)^2$$

Now, the highest power of $\frac{dy}{dx}$ is **2**,

so **Degree** = **2**

✅ **Final Answer:**

Order = **1**, **Degree** = **2**

Question320

The order of the differential equation whose solution is $y = a\cos x + b\sin x + ce^{-x}$, is MHT CET 2007

Options:

A. 3

B. 1

C. 2

D. 4

Answer: A

Solution:

In given equation their are three parameters, so its differential equation is third order differential equation.

Question321

The differential equation of all parabolas with axis parallel to the axis of y is MHT CET 2007

Options:

A. $y_2 = 2y_1 + x$

B. $y_3 = 2y_1$

C. $y_2^3 = y_1$

D. None of these

Answer: D

Solution:

Step 1: General equation of a parabola with axis parallel to y-axis

The general form is:

$$(x-h)^2 = 4a(y-k)$$

where h, k, a are constants.

Step 2: Differentiate with respect to x

$$2(x-h) = 4a\frac{dy}{dx}$$

$$\Rightarrow x-h = 2a\frac{dy}{dx} \quad \text{...(i)}$$

Step 3: Differentiate again

Differentiate equation (i) again w.r.t x:

$$1 = 2a\frac{d^2y}{dx^2}$$

$$\Rightarrow a = \frac{1}{2\frac{d^2y}{dx^2}} \quad \text{...(ii)}$$

Step 4: Substitute value of a from (ii) into (i)

$$x-h = 2 \times \frac{1}{2\frac{d^2y}{dx^2}}\frac{dy}{dx}$$

$$\Rightarrow x-h = \frac{(dy/dx)}{(d^2y/dx^2)}$$

Step 5: Eliminate h

Differentiate again to remove h:

$$1 = \frac{(d^2y/dx^2)^2 - (dy/dx)(d^3y/dx^3)}{(d^2y/dx^2)^2}$$

Simplify:

$$(d^2y/dx^2)^2 = (dy/dx)(d^3y/dx^3)$$

✅ **Final Differential Equation:**

$$y_1y_3 = (y_2)^2$$

where
$y_1 = \frac{dy}{dx}$, $y_2 = \frac{d^2y}{dx^2}$, $y_3 = \frac{d^3y}{dx^3}$

Therefore, the correct answer is — "None of these."
